Lezione 12

1

S Retta che contiene il raggio della circonferenza

S asse della corda AB

$s \perp \overline{AB}$ $\overline{AM} = \overline{MB}$

2

$d(C, r) = R$

DISTANZA CENTRO - RETTA TANGENTE

RETTA

3

ESTERNA ($\Delta < 0$)

TANGENTE ($\Delta = 0$)

SECANTE ($\Delta > 0$)

$$\begin{cases} y - y_0 = m(x - x_0) \\ x^2 + y^2 + \alpha x + \beta y + \gamma = 0 \end{cases}$$

$\alpha, \beta, \gamma \in \mathbb{R}$

$(x_0, y_0) \in \mathbb{R}$

$r: y = 6mx - 5 \qquad m \in \mathbb{R} : C \in r$

C CENTRO DI

$\Gamma : \underline{x^2 + y^2 - 6x - 10y - 2 = 0}$

$C\left(-\frac{\alpha}{2}; -\frac{\beta}{2}\right) = \left(-\frac{-6}{2}; -\frac{-10}{2}\right) = (3;5)$

$C(3;5) \qquad C \qquad 5 = 6m \cdot 3 - 5 \qquad 10 = 18m \Rightarrow 18m = 10$

$m = \frac{\cancel{10}\,5}{\cancel{18}\,9}$

$\boxed{m = \frac{5}{9}}$

$$\Gamma: \begin{cases} x^2 + y^2 - 3x + 4y - 2 = 0 \\ y = mx + 1 \end{cases}$$

$m \in r : \exists\ r$: tangente a Γ

$$x^2 + (mx+1)^2 - 3x + 4(mx+1) - 2 = 0$$

$$x^2 + m^2x^2 + 1 + 2mx - 3x + 4mx + 4 - 2 = 0$$

$$(1+m^2)x^2 + 6mx - 3x + 3 = 0$$

$$\boxed{(1+m^2)x^2 + 3(2m-1)x + 3 = 0}$$

$a = 1 + m^2$

$b = 3(2m-1)$

$c = 3$

$\boxed{\Delta = 0}$

CONDIZ. TANGENZA

$$\Delta = b^2 - 4ac = 0$$

$$\Delta = [3(2m-1)]^2 - 4(1+m^2)\cdot 3 = 0$$

$$9(2m-1)^2 - 12 - 12m^2 = 0$$

$$9(4m^2 + 1 - 4m) - 12 - 12m^2 = 0$$

$$36m^2 + 9 - 36m - 12 - 12m^2 = 0$$

$$24m^2 - 36m - 3 = 0$$

$$8m^2 - 12m - 1 = 0$$

$$\Delta = 144 - 4(8)(-1) = 144 + 32 = 176$$

$$m_1, m_2 = \frac{12 \pm \sqrt{176}}{16} =$$

$$= \frac{12 \pm 4\sqrt{11}}{16} = \frac{\cancel{4}(3 \pm \sqrt{11})}{\cancel{16}\,4} =$$

$$= \frac{3 \pm \sqrt{11}}{4}$$

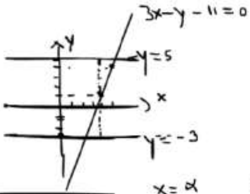
$$\boxed{m = \frac{3 \pm \sqrt{11}}{4}}$$

176	2
88	2
44	2
22	2
11	11
1	

$$\sqrt{176} = \sqrt{2^4 \cdot 11}$$

$$= 2^2\sqrt{11}$$

$$= 4\sqrt{11}$$



$x=\alpha$

$\boxed{y=3x-11}$

$\boxed{c(\alpha;\,3\alpha-11)}$

$c\in r: y+11=3x$

$1^{a}\,tg: y+3=0\Rightarrow y=-3$

$2^{a}\,tg: y-5=0\Rightarrow y=5$

$3x-y-11=0$

x	y
4	1
5	4

$$\underline{d(c,\,1^{a}\,tg)}=\underline{d(c,\,2^{a}\,tg)}=R$$

$$\frac{|ax_c+by_c+c|}{\sqrt{a^2+b^2}}=\frac{|a'x_c+b'y_c+c'|}{\sqrt{a'^2+b'^2}}$$

$a=0\quad b=1\quad c=3$ $\qquad a'=0\quad b'=1\quad c'=-5$

$x_c=\alpha\quad y_c=3\alpha-11$ $\qquad x_c=\alpha\quad y_c=3\alpha-11$

$$|3\alpha-11+3|=|3\alpha-11-5|$$

$3\cancel{\alpha}-\cancel{11}+3=\cancel{3\alpha}-\cancel{11}-5$

$3=-5$

$\cancel{\nexists}\ x\in\mathbb{R}$

$3\alpha-11+3=-3\alpha+11+5$

$6\alpha=22+5-3$

$6\alpha=24\Longrightarrow\alpha=\frac{\cancel{24}^{4}}{\cancel{6}_{1}}=4$

$\alpha=4\qquad c(\alpha;\,3\alpha-11)=(4;\,3\cdot4-11)=(4;\,1)$

$c(4;\,1)\qquad R=4$

$(x-x_c)^2+(y-y_c)^2=R^2$

$(x-4)^2+(y-1)^2=16$

$x^2+\cancel{16}-8x+y^2+1-2y=\cancel{16}$

$$\boxed{x^2+y^2-8x-2y+1=0}$$

$x^2+y^2+\alpha x+\beta y+\gamma=0$

$A(2;1)$ $\underline{B(1;3)}$ $\boxed{C_{\gamma}\ y=2x+1}$

$A(2;1)\quad 4+1+2\alpha+\beta+\gamma=0$

$B(1;3)\quad 1+9+\alpha+3\beta+\gamma=0$

$$\begin{cases} 2\alpha+\beta+\gamma=-5 \\ \alpha+3\beta+\gamma=-10 \end{cases}$$

$$\begin{cases} \alpha=-3\beta-\gamma-10 \\ 2(-3\beta-\gamma-10)+\beta+\gamma=-5 \end{cases}$$

$$\begin{cases} \alpha=-3\beta-\gamma-10 \\ -6\beta-2\gamma-20+\beta+\gamma=-5 \end{cases}$$

$$\begin{cases} -5\beta-\gamma=15 \end{cases} \Longrightarrow 5\beta+\gamma=-15 \Longrightarrow \boxed{\gamma=-15-5\beta}$$

$\alpha=-3\beta-(-15-5\beta)-10=$

$=-3\beta+15+5\beta-10=2\beta+5$

$\alpha=2\beta+5$

$\gamma=-15-5\beta$

$$\begin{cases} x^2+y^2+(2\beta+5)x+\beta y-15-5\beta=0 \\ y=2x+1 \end{cases}$$

$x^2+4x^2+1+4x+(2\beta+5)x+\beta(2x+1)-15-5\beta=0$

$5x^2+(4x+2\beta x+5x+2\beta x)+1+\beta-15-5\beta=0$

$5x^2+(9+4\beta)x-14-4\beta=0$

$\Delta=(9+4\beta)^2-20(-14-4\beta)=0$

$\Delta=81+16\beta^2+72\beta+280+80\beta=0$

$16\beta^2+152\beta+361=0$

$\Delta=(152)^2-4(16)(361)$

$=23104-23104=0$

$\beta_1=\beta_2=-\frac{\cancel{152}^{19}}{\cancel{32}_{4}}=-\frac{19}{4}$

$x^2+y^2+\left[\cancel{2}^{1}\left(-\frac{19}{\cancel{4}_{2}}\right)+5\right]x+\left(-\frac{19}{4}\right)y-15-5\left(-\frac{19}{4}\right)=0$

$x^2+y^2+\left[\frac{-19+10}{2}\right]x-\frac{19}{4}y-15+\frac{95}{4}=0$

$x^2+y^2-\frac{9}{2}x-\frac{19}{4}y+\frac{-60+95}{4}=0$

$$\boxed{x^2+y^2-\frac{9}{2}x-\frac{19}{4}y-\frac{35}{4}=0}$$