Lecture 01

Circonférence passe par trois points

A(-3;3) B(-2;4) C(1;5) $\alpha, \beta, \gamma = ?$

$x^2 + y^2 + \alpha x + \beta y + \gamma = 0$

A(-3;3) $9 + 9 - 3\alpha + 3\beta + \gamma = 0$

B(-2;4) $4 + 16 - 2\alpha + 4\beta + \gamma = 0$

C(1;5) $1 + 25 + \alpha + 5\beta + \gamma = 0$

$\begin{cases} -3\alpha + 3\beta + \gamma = -18 \\ -2\alpha + 4\beta + \gamma = -20 \\ \alpha + 5\beta + \gamma = -26 \end{cases} \Rightarrow \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix} \begin{cases} 3\alpha - 3\beta - \gamma = 18 \\ 2\alpha - 4\beta - \gamma = 20 \\ \alpha + 5\beta + \gamma = -26 \end{cases}$

$\Rightarrow \begin{matrix} R_1 - R_2 \\ R_2 \\ R_3 \end{matrix} \begin{cases} \alpha + \beta = -2 \\ 2\alpha - 4\beta - \gamma = 20 \\ \alpha + 5\beta + \gamma = -26 \end{cases} \Rightarrow \begin{matrix} R_1 - R_2 \\ R_2 + R_3 \\ R_3 \end{matrix} \begin{cases} \alpha + \beta = -2 \\ 3\alpha + \beta = -6 \\ \alpha + 5\beta + \gamma = -26 \end{cases}$

$\begin{matrix} R'_1 \\ R'_2 \end{matrix} \begin{cases} \alpha + \beta = -2 \\ 3\alpha + \beta = -6 \end{cases}$

$R'_1 - R'_2 \quad -2\alpha = 4 \Rightarrow \alpha = -\frac{4}{2} \Rightarrow \boxed{\alpha = -2}$

$\alpha + \beta = -2 \quad -2 + \beta = -2 \Rightarrow \boxed{\beta = 0}$

$\alpha + \beta + \gamma = -26$

$-2 + \gamma = -26 \Rightarrow \boxed{\gamma = -24}$

$\begin{cases} \alpha = -2 \\ \beta = 0 \\ \gamma = -24 \end{cases}$ $\boxed{x^2 + y^2 - 2x - 24 = 0}$

$C\left(-\frac{\alpha}{2}; -\frac{\beta}{2}\right) \quad r = \sqrt{\left(\frac{\alpha}{2}\right)^2 + \left(\frac{\beta}{2}\right)^2 - \gamma}$

$C\left(+\frac{2}{2}; 0\right) \quad C(1;0)$

$r = \sqrt{\frac{1}{4} - (-24)} = \sqrt{\frac{1}{4} + 24} = \sqrt{\frac{1 + 96}{4}} = \frac{\sqrt{97}}{2}$

$\boxed{r = \frac{\sqrt{97}}{2}}$

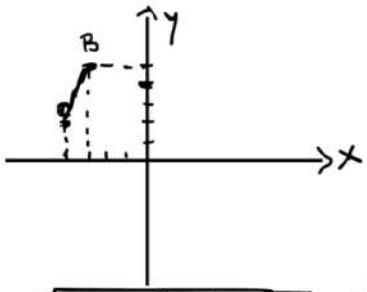
$(x - x_0)^2 + (y - y_0)^2 = r^2$

$\boxed{(x-1)^2 + y^2 = \frac{97}{4}}$

(T) : ?

C (−4; 2)

B (−3; 5)



$R = \overline{BC} = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{(-4+3)^2 + (2-5)^2} = \sqrt{1+9}$

$\boxed{R = \sqrt{10}}$

C (−4; 2) $R = \sqrt{10}$

$(x - x_C)^2 + (y - y_C)^2 = R^2$

$(x+4)^2 + (y-2)^2 = 10$

$x^2 + 16 + 8x + y^2 + 4 - 4y = 10$

$\boxed{x^2 + y^2 + 8x - 4y + 10 = 0}$

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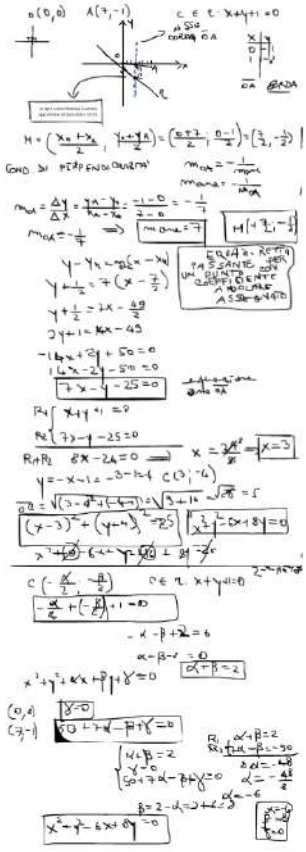


$C(4;2)$

$r: y = 3x+2$

x	y
0	2
1	5

$x_c = 4 \quad y_c = 2$

$-3x + y - 2 = 0$

$3x - y + 2 = 0$

$a = 3$

$b = -1$

$c = 2$

$$d(C,r) = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2+b^2}}$$

$$d(C,r) = \frac{|3\cdot 4 + (-1)\cdot 2 + 2|}{\sqrt{9+1}}$$

$$R = d(C,r) = \frac{|12 - 2 + 2|}{\sqrt{10}} = \frac{12}{\sqrt{10}}$$

$$\boxed{R = \frac{12}{\sqrt{10}}}$$

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$(x-4)^2 + (y-2)^2 = \frac{144}{10}$$

$$x^2 + 16 - 8x + y^2 + 4 - 4y = \frac{144}{10}$$

$$10x^2 + 10y^2 - 80x - 40y + 200 = 144$$

$$10x^2 + 10y^2 - 80x - 40y + 56 = 0$$

$$\boxed{5x^2 - 5y^2 - 40x - 20y + 28 = 0}$$

METODO 2

$$x^2 + y^2 + ax + by + c = 0$$

$C(4;2) = \left(-\frac{a}{2}; -\frac{b}{2}\right)$

$\boxed{a = -8 \quad b = -4}$

$$\begin{cases} x^2 + y^2 - 8x - 4y + \gamma = 0 \\ y = 3x + 2 \end{cases}$$

$\Delta(\gamma) = 0$

CONDIZIONE TANGENZA

$$x^2 + (3x+2)^2 - 8x - 4(3x+2) + \gamma = 0$$

$$x^2 + 9x^2 + 4 + 12x - 8x - 12x - 8 + \gamma = 0$$

$$10x^2 - 8x - 4 + \gamma = 0$$

$\Delta(\gamma) = 0 \quad \Delta(\gamma) = a_4 - 4(10)(-4+\gamma) = 0$

$$64 + 160 - 40\gamma = 0$$

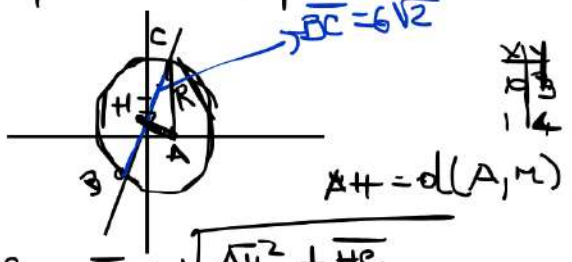
$$224 - 40\gamma = 0 \Rightarrow \gamma = \frac{224}{40} = \frac{56}{10} = \frac{28}{5}$$

$$\gamma = \frac{28}{5}$$

$$x^2 + y^2 - 8x - 4y + \frac{28}{5} = 0$$

$$\boxed{5x^2 + 5y^2 - 40x - 20y + 28 = 0}$$

$A(2;0)$ $\quad d: y = x + 3$ $\quad \overline{BC} = 6\sqrt{2}$



$\overline{BC} = 6\sqrt{2}$

$AH = d(A, m)$

$\overline{HC} = \frac{\overline{BC}}{2}$ $\quad R = \overline{AC} = \sqrt{\overline{AH}^2 + \overline{HC}}$