## Lezione 22

$\boxed{\log x = 3}$

In un'equazione logaritmica gli aspetti da attenzionare sono:
-l'incognita figura nell'argomento del logaritmo
-l'argomento va discusso e deve essere posto sempre maggiore di 0
-Come faccio a risolverla?

$\boxed{\log_e x = 3}$

Per risolvere l'equazione devo trasformare il 3 in un logaritmo. COME?!!!!?

$\boxed{\log_e e = 1}$

$\log_e e^3 = 3 \log_e e$

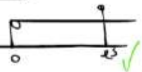
$\boxed{\log_e x = \log_e e^3}$

Due logaritmi nella stessa base sono uguali quando gli argomenti sono uguali

$\begin{cases} \boxed{x = e^3} & \text{Risoluzione} \\ \boxed{x > 0} & \text{Condiz. argomento} \end{cases}$

$x = e^3$

<u>accettabile</u>




$\ln = \log_e$

ES $\ln \underline{x} = 2 \ln \underline{2x}$

$\begin{cases} x > 0 & 1^a \text{ ARG.} \\ 2x > 0 & 2^a \text{ ARG.} \\ \ln x = 2 \ln 2x \end{cases}$

$\boxed{a \log b = \log b^a}$

$\ln x = \ln (2x)^2$

$\Downarrow$

$x = (2x)^2$

$x = 4x^2$

$-4x^2 + x = 0 \Rightarrow 4x^2 - x = 0$

$x(4x-1) = 0$

$\underline{x = 0}$

$4x - 1 = 0$

$\underline{x = \frac{1}{4}}$

$\begin{cases} x > 0 \\ x > 0 \\ \boxed{x = 0} \end{cases} \vee \boxed{x = \frac{1}{4}}$

NO !!!

$\ln(28-x^3) - \ln(7-x) = 2\ln(4-x)$

$$\begin{cases} 28-x^3>0 \\ 7-x>0 \\ 4-x>0 \\ \ln(28-x^3)-\ln(7-x)=2\ln(4-x) \end{cases}$$

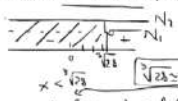
$$\begin{cases} 28-x^3>0 \Longrightarrow x^3-28<0 \Longrightarrow \boxed{x<\sqrt[3]{28}} \\ 7-x>0 \Longrightarrow -x>-7 \Longrightarrow \boxed{x<7} \\ 4-x>0 \Longrightarrow -x>-4 \Longrightarrow \boxed{x<4} \end{cases}$$

DIFF. DI CUBI

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$x^3-28=\underbrace{(x-\sqrt[3]{28})}(x^2+\sqrt[3]{28}\,x+\sqrt[3]{(28)^2}) \rightarrow \Delta<0$

$(x-\sqrt[3]{28})(x^2+\sqrt[3]{28}\,x+\sqrt[3]{(28)^2})<0$



$N_1$

$N_2$ $x^2+\sqrt[3]{28}\,x+\sqrt[3]{28^2}>0$ $\Delta<0$ $\forall x\in\mathbb{R}$

$N_1$: $x-\sqrt[3]{28}>0$ $x>\sqrt[3]{28}$

$\boxed{\sqrt[3]{28}\simeq 3{,}03}$

$x<\sqrt[3]{28}$

$\ln(28-x^3)-\ln(7-x)=2\ln(4-x)$

$\boxed{\ln\frac{28-x^3}{7-x}=\ln(4-x)^2}$

$\boxed{\ln a-\ln b=\ln\frac{a}{b}}$

$\boxed{a\ln b=\ln b^a}$

$\frac{28-x^3}{7-x}=(4-x)^2$

$\frac{28-x^3}{\cancel{7-x}}=\frac{(4-x)^2(7-x)}{\cancel{7-x}}$

$28-x^3=(16+x^2-8x)(7-x)$

$28-\cancel{x^3}=112-16x+7x^2-\cancel{x^3}-56x+8x^2$

$-15x^2+72x-84=0$

$15x^2-72x+84=0$

$5x^2-24x+28=0$

$\Delta=b^2-4ac=(-24)^2-4(5)(28)=$

$=576-560=16$

$x_1,x_2=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{24\pm4}{10}=$ $\frac{20}{10}=\boxed{2}$ $\frac{28}{10}=\boxed{\frac{14}{5}}$

$\sqrt[3]{28}\simeq 3{,}03$

$$\begin{cases} x<\sqrt[3]{28} \\ x<7 \\ x<4 \\ \boxed{x_1=2 \quad x_2=\frac{14}{5}=2{,}8} \end{cases}$$



$\frac{14}{5}$

$14 \mid 5$
$10 \mid 2{,}8$
$40$

$$\log_{\frac{1}{2}}(x-1) - \log_2(x+1) = 3$$

$$\begin{cases} x-1 > 0 \Longrightarrow \boxed{x>1} \\ x+1 > 0 \Longrightarrow \boxed{x>-1} \\ \log_{\frac{1}{2}}(x-1) - \log_2(x+1) = 3 \end{cases}$$

$$\boxed{\log_c b = \frac{\log_a b}{\log_a c}}$$

FORMULA DEL CAMB. DI BASE

$c = \frac{1}{2}$

$b = x-1$

$a = 2$

$$\boxed{\log_{\frac{1}{2}}(x-1) = \frac{\log_2(x-1)}{\log_2 \frac{1}{2}} = -\log_2(x-1)}$$

$$-\log_2(x-1) - \log_2(x+1) = 3$$

$$\log_2(x-1) + \log_2(x+1) = -3$$

$$\log_2[(x-1)(x+1)] = -3$$

$$\boxed{\log_2 a + \log_2 b = \log_2 a \cdot b}$$

$$a \log b = \log b^a$$

$$\log_2(x^2-1) = \log_2 2^{-3}$$

$$x^2 - 1 = 2^{-3}$$

$$x^2 - 1 = \frac{1}{8}$$

$$x^2 = 1 + \frac{1}{8} = \frac{8+1}{8} = \frac{9}{8}$$

$$\boxed{x^2 = \frac{9}{8}}$$

$$x = \pm\sqrt{\frac{9}{8}}$$

$$\begin{array}{r|l} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

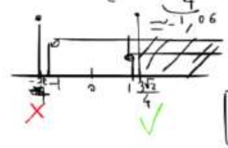
$$\sqrt{8} = \sqrt{2^3} = \sqrt{2 \cdot 2^2} = 2\sqrt{2}$$

$$x = \pm\frac{\sqrt{9}}{\sqrt{8}} = \pm\frac{3}{\sqrt{8}}$$

$$= \pm\frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{3\sqrt{2}}{4}$$

$$x = \pm\frac{3\sqrt{2}}{4}$$

$$\begin{cases} x > 1 \\ x > -1 \\ x_1 = -\underbrace{\frac{3\sqrt{2}}{4}}_{\simeq -1,06} \vee x_2 = \underbrace{\frac{3\sqrt{2}}{4}}_{\simeq 1,06} \end{cases}$$




$$\boxed{x = \frac{3\sqrt{2}}{4}} \checkmark$$

$$\log_3^2(29-2)+\log_{\frac{1}{3}}(29-2)=6$$

$$\log_3^2(27)+\log_{\frac{1}{3}}(27)=6$$

$$3^2+(-3)= \checkmark$$

$$=9-3=6$$

$$\boxed{\log_3^2(x-2)+\log_{\frac{1}{3}}(x-2)=6}$$

$$\log_c b=\frac{\log_a b}{\log_a c}$$

$c=\frac{1}{3}$

$b=x-2$

$a=3$

$$\boxed{\log_{\frac{1}{3}}(x-2)=\frac{\log_3(x-2)}{\log_3\frac{1}{3}}=-\log_3(x-2)}$$

$$\boxed{\log_3^2(x-2)-\log_3(x-2)-6=0}$$

$$\begin{cases} x-2>0 \Rightarrow \boxed{x>2} \\ \boxed{\log_3^2(x-2)-\log_3(x-2)-6=0} \end{cases}$$

$$\boxed{\log_3(x-2)=t}$$

$$t^2-t-6=0$$

$$\Delta=b^2-4ac=(-1)^2-4(1)(-6)=1+24=25$$

$$t_1,t_2=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{1\pm5}{2}$$

$$\frac{\cancel{4}}{\cancel{2}}^{2}=-2$$

$$\frac{\cancel{6}}{\cancel{2}}^{3}=3$$

$$t_1=-2 \quad t_2=3$$

$$\log_3(x-2)=-2 \Rightarrow \log_3(x-2)=\log_3 3^{-2}$$

$$x-2=\frac{1}{9} \Rightarrow x=\frac{1}{9}+2=\frac{1+18}{9}$$

$$\boxed{x=\frac{19}{9}}$$

$$\log_3(x-2)=3 \Rightarrow \log_3(x-2)=\log_3 3^3$$

$$x-2=27 \Rightarrow \boxed{x=29}$$

$$\begin{cases} x>2 \\ x_1=\frac{19}{9}\ ;\ x_2=29 \end{cases}$$

$$\boxed{\frac{3}{\log_2 x - 1} + \frac{2}{\log_2 x + 1} = 2}$$

C.E.

$$\begin{cases} x > 0 \\ Ⓐ\ \log_2 x - 1 \neq 0 \\ Ⓑ\ \log_2 x + 1 \neq 0 \end{cases}$$

Ⓐ $\log_2 x - 1 \neq 0$

$\log_2 x \neq 1 \Rightarrow \log_2 x \neq \log_2 2^1$

$\boxed{x \neq 2}$

Ⓑ $\log_2 x + 1 \neq 0$

$\log_2 x \neq -1 \Rightarrow \log_2 x \neq \log_2 2^{-1}$

$\boxed{x \neq \frac{1}{2}}$

$$\begin{cases} x > 0 \\ x \neq 2 \\ x \neq \frac{1}{2} \end{cases}$$

$t = \log_2 x$

$$\frac{3}{t-1} + \frac{2}{t+1} = 2$$

$$\frac{3(t+1)+2(t-1)}{\cancel{(t-1)(t+1)}} = \frac{2(t^2-1)}{\cancel{(t-1)(t+1)}} \quad \boxed{t \neq 1}$$

$$3t + 3 + 2t \cancel{-2} = 2t^2 \cancel{-2}$$

$$-2t^2 + 5t + 3 = 0$$

$$2t^2 - 5t - 3 = 0$$

$$\Delta = b^2 - 4ac = 25 - 4(2)(-3) = 25 + 24 = 49$$

$$t_1, t_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{5 \pm 7}{4} \begin{cases} \frac{-2}{4} = -\frac{1}{2} \\ \frac{12}{4} = 3 \end{cases}$$

$t_1 = -\frac{1}{2}$ ; $t_2 = 3$

$$\log_2 x = -\frac{1}{2} \Longrightarrow \log_2 x = \log_2 2^{-\frac{1}{2}}$$

$$x = 2^{-\frac{1}{2}} = \frac{1}{2^{\frac{1}{2}}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$$

$$\log_2 x = 3 \Longrightarrow \log_2 x = \log_2 2^3$$

$$x = 2^3 = \boxed{8}$$

$x_1 = \frac{\sqrt{2}}{2}$ ; $x_2 = 8$

✓ ✓

$$\begin{cases} x > 0 \\ x \neq 2 \\ x \neq +\frac{1}{2} \end{cases}$$