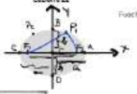
Lezione 22



$P(x;y)$

Fuochi $\begin{cases} F_1(-c,0) \\ F_2(c,0) \end{cases}$

$C(0,0)$

Centro di simmetria

Asse x, Asse y

$y=0$, $x=0$

Assi di simmetria

$$\boxed{\overline{PF_1}+\overline{PF_2}=2a}$$

$2a = AC$ Asse maggiore

$2b = BD$ Asse minore

$a > b$

I fuochi stanno sul semiasse maggiore

$$\overline{PF_1}=\sqrt{(-c-x)^2+(0-y)^2}$$
$$\overline{PF_2}=\sqrt{(c-x)^2+(0-y)^2}$$

$$\sqrt{(-c-x)^2+(0-y)^2}+\sqrt{(c-x)^2+(0-y)^2}=2a$$

$$\sqrt{(-c-x)^2+(0-y)^2}=2a-\sqrt{(c-x)^2+(0-y)^2}$$

$$(-c-x)^2+(0-y)^2=4a^2+(c-x)^2+(0-y)^2-4a\sqrt{(c-x)^2+(0-y)^2}$$

$$\cancel{c^2}+\cancel{x^2}+2cx+\cancel{y^2}=4a^2+\cancel{c^2}+\cancel{x^2}-2cx+\cancel{y^2}-4a\sqrt{(c-x)^2+(0-y)^2}$$

$$4cx-4a^2=-4a\sqrt{(c-x)^2+(0-y)^2}$$

$$a^2-cx=a\sqrt{(c-x)^2+(0-y)^2}$$

$$a^4+c^2x^2-2a^2cx=a^2\left[(c-x)^2+y^2\right]$$

$$a^4+c^2x^2-2a^2cx=a^2\left[c^2+x^2-2cx+y^2\right]$$

$$a^4+c^2x^2\cancel{-2a^2cx}=a^2c^2+a^2x^2\cancel{-2a^2cx}+a^2y^2$$

$$c^2x^2-a^2x^2-a^2y^2=a^2c^2-a^4$$

$$(c^2-a^2)x^2-a^2y^2=a^2(c^2-a^2)$$

$$(a^2-c^2)x^2+a^2y^2=a^2(a^2-c^2)$$

$$\boxed{a^2-c^2=b^2}$$

$$x^2+\frac{a^2}{a^2-c^2}y^2=a^2$$

$$x^2+\frac{a^2}{b^2}y^2=a^2$$

$$\frac{x^2}{a^2}+\frac{\cancel{a^2}}{b^2\cancel{a^2}}y^2=1 \implies \boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$$

$$\boxed{\overline{PF_1}+\overline{PF_2}=2b}$$

$F_1(-c,0)$

$F_2(c;0)$

$D(0;b)$

$b > a$

$$\boxed{a^2=b^2-c^2}$$

$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1}$$

EQ CANONICA

$\boxed{PF_1 + PF_2 = \underbrace{10}_{\substack{2a \\ 2b}}}$

$10 = 2a$

$10 = 2b$

?!?

$\boxed{A(3;1)}$

a) Fuochi sull'asse delle x

$2a = 10 \Rightarrow a = \frac{\cancel{10}\,5}{\cancel{2}\,1} \Longrightarrow a = 5$

$a = 5 \qquad P(3;1)$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \boxed{\frac{x^2}{25} + \frac{y^2}{b^2} = 1}$$

$$P(3;1) \Rightarrow \frac{9}{25} + \frac{1}{b^2} = 1$$

$$\frac{9b^2}{25b^2} + \frac{25}{25b^2} = \frac{25b^2}{25b^2}$$

$$-16b^2 = -25 \Rightarrow \boxed{b^2 = \frac{25}{16}}$$

$$\frac{x^2}{25} + \frac{y^2}{\frac{25}{16}} = 1$$

$$\frac{x^2}{25} + \frac{16y^2}{25} = 1 \Rightarrow \boxed{x^2 + 16y^2 = 25}$$

b) Fuochi sull'asse delle y

$\boxed{b = 5} \qquad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

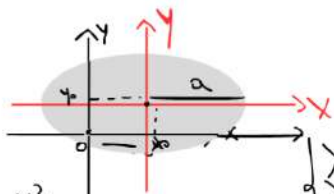
$$\frac{x^2}{a^2} + \frac{y^2}{25} = 1$$

$$\frac{9}{a^2} + \frac{1}{25} = 1 \qquad \frac{225 + a^2}{25a^2} = \frac{25a^2}{25a^2}$$

$$-24a^2 = -225 \qquad a^2 = \frac{\cancel{225}\,75}{\cancel{24}\,8} \qquad \boxed{a^2 = \frac{75}{8}}$$

$$\frac{x^2}{\frac{75}{8}} + \frac{y^2}{25} = 1 \qquad \frac{8x^2}{75} + \frac{y^2}{25} = 1$$

$$\boxed{8x^2 + 3y^2 = 75}$$



$O(0;0)$
$C(x_0, y_0)$

$$\begin{cases} X = x - x_0 \\ Y = y - y_0 \end{cases}$$

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1$$

$$\boxed{\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1}$$

$$\frac{x^2 + x_0^2 - 2x_0 x}{a^2} + \frac{y^2 + y_0^2 - 2y_0 y}{b^2} = 1$$

$$\frac{b^2 x^2 + b^2 x_0^2 - 2b^2 x_0 x + a^2 y^2 + a^2 y_0^2 - 2a^2 y_0 y}{\cancel{a^2 b^2}} = \frac{a^2 b^2}{\cancel{a^2 b^2}}$$

$$\underbrace{b^2 x^2}_{m} + \underbrace{a^2 y^2}_{n} \underbrace{- 2 b^2 x_0 x}_{p} \underbrace{- 2a^2 y_0 y}_{q} + \underbrace{b^2 x_0^2 + a^2 y_0^2 - a^2 b^2}_{r} = 0$$

$$\boxed{m x^2 + n y^2 + p x + q y + r = 0}$$

$m = b^2$
$n = a^2$
$p = -2b^2 x_0$
$q = -2a^2 y_0$
$r = b^2 x_0^2 + a^2 y_0^2 - a^2 b^2$

Equazione ellisse in forma implicita

$$4x^2 + 2y^2 - 4x + 12y + 11 = 0$$

$$4(\underbrace{x^2}_{\text{QUADRATO DI } x} - \underbrace{x}_{\text{DOPPIO PRODOTTO } \frac{1}{2}\cdot x}) + 2(\underbrace{y^2}_{\text{QUADRATO DI } y} + \underbrace{6y}_{\text{DOPPIO PRODOTTO } y\cdot 3}) + 11 = 0 \qquad \boxed{\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1}$$

$$4\left[\left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4}\right] + 2\left[\left(y^2 + 6y + 9\right) - 9\right] + 11 = 0$$

$$4\left[\left(x - \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 2\left[\left(y+3\right)^2 - 9\right] + 11$$

$$4\left(x - \frac{1}{2}\right)^2 - 1 + 2\left(y+3\right)^2 - 18 + 11 = 0$$

$$4\left(x - \frac{1}{2}\right)^2 + 2\left(y+3\right)^2 = 8$$

$$\boxed{\frac{\left(x - \frac{1}{2}\right)^2}{2} + \frac{\left(y+3\right)^2}{4} = 1}$$

$c\left(\frac{1}{2} ; 3\right)$ $\qquad a^2 = 2 \Rightarrow a = \sqrt{2}$

$\qquad\qquad\qquad b^2 = 4 \Rightarrow b = \sqrt{4} = 2$

$$x^2 + 9y^2 - 2x + 36y + 46 = 0$$

$$(x^2 - 2x) + 9(y^2 + 4y) + 46 = 0$$

$$\left[(x^2 - 2x + 1) - 1\right] + 9\left[(y^2 + 4y + 4) - 4\right] + 46 = 0$$

$$(x-1)^2 - 1 + 9(y+2)^2 - 36 + 46 = 0$$

$$(x-1)^2 + 9(y+2)^2 = -9$$

$$\boxed{\frac{(x-1)^2}{9} + \frac{(y+2)^2}{1} = -1}$$

?!?!

È ELLISSE IMMAGINARIO

$\sqrt{-1} = i$

Eccentricità

$$e = \frac{c}{a} < 1$$

$c < a$

c SEMIDIST. FOCALE

a LUNGHEZZA SEMIASSE

$\boxed{e = 0}$ $c = 0$ NELLA CIRCONFERENZA