Lezione 23

$$\frac{1}{2\log_2 x - 2} + \frac{3}{\log_2^2 x - 1} = \frac{1}{4}$$

$$\frac{1}{2(\log_2 x - 1)} + \frac{3}{(\log_2 x + 1)(\log_2 x - 1)} = \frac{1}{4}$$

$$\frac{2(\log_2 x + 1) + 12}{4(\log_2 x + 1)(\log_2 x - 1)} = \frac{\log_2^2 x - 1}{4(\log_2 x + 1)(\log_2 x - 1)}$$

$$C.E. \begin{cases} \log_2 x + 1 \neq 0 \Rightarrow \log_2 x \neq -1 \\ \qquad \log_2 x \neq \log_2 2^{-1} \Rightarrow x \neq 2^{-\frac{1}{2}} \\ \qquad x \neq \frac{1}{2} \\ \log_2 x - 1 \neq 0 \Rightarrow \log_2 x \neq 1 \\ \qquad \log_2 x \neq \log_2 2 \Rightarrow \boxed{x \neq 2} \\ x > 0 \end{cases}$$

$$2\log_2 x + 2 + 12 = \log_2^2 x - 1$$

$$-\log_2^2 x + 2\log_2 x + 15 = 0 \qquad \log_2 x = t$$

$$\log_2^2 x - 2\log_2 x - 15 = 0$$

$$t^2 - 2t - 15 = 0$$

$$\Delta = b^2 - 4ac = (-2)^2 - 4(1)(-15) = 4 + 60 = 64$$

$$t_1, t_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{2 \pm \sqrt{64}}{2} \begin{cases} \frac{2-8}{2} = -\frac{6}{2} = -3 \\ \frac{2+8}{2} = \frac{10}{2} = 5 \end{cases}$$

$$t_1 = -3 \; ; \; t_2 = 5$$

$$\log_2 x = -3 \Rightarrow \log_2 x = \log_2 2^{-3} \Rightarrow x = 2^{-3} \quad \boxed{x = \frac{1}{8}}$$

$$\log_2 x = 5 \Rightarrow \log_2 x = \log_2 2^5 \quad \boxed{x = 32}$$

$$\boxed{x_1 = \frac{1}{8} \vee x = 32}$$

$$2\log_2(x+3)+\frac{2}{\log_2(x+3)}=5$$

$$\frac{2\log_2^2(x+3)+2}{\log_2(x+3)}=\frac{5\log_2(x+3)}{\log_2(x+3)}$$

$$C.E\begin{cases}\log_2(x+3)\neq 0\Longrightarrow \log_2\boxed{(x+3)}\neq \log_2 \boxed{2^0}\Rightarrow x+3\neq 1 & \boxed{x\neq -2}\\ x+3>0\Longrightarrow \boxed{x>-3}\end{cases}$$

$$2\log_2^2(x+3)-5\log_2(x+3)+2=0$$

$\log_2(x+3)=t$

$$2t^2-5t+2=0$$

$$\Delta=b^2-4ac=(-5)^2-4\cdot(2)(2)=25-16=9$$

$$t_1,t_2=-\frac{b\pm\sqrt{\Delta}}{2a}=\frac{5\pm3}{4}\;\begin{cases}\frac{5-3}{4}=\frac{\cancel{2}^{1}}{\cancel{4}_{2}}=\boxed{\frac{1}{2}}\\ \frac{5+3}{4}=\frac{\cancel{8}^{2}}{\cancel{4}_{1}}=\boxed{2}\end{cases}$$

$$t_1=\frac{1}{2}\ \vee\ t_2=2$$

$$\log_2(x+3)=\frac{1}{2}\Longrightarrow \log_2(x+3)=\log_2 2^{\frac{1}{2}}\Longrightarrow x+3=2^{\frac{1}{2}}$$

$$\boxed{x=\sqrt{2}-3}$$

$2^{\frac{1}{2}}=\sqrt{2}$

$$\log_2(x+3)=2\Longrightarrow \log_2(x+3)=\log_2 2^2$$

$$x+3=4\Longrightarrow \boxed{x=1}$$

$$\boxed{x_1=\sqrt{2}-3\ \vee\ x_2=1}$$

$$\frac{3}{\log_4 x} + \frac{3}{2+\log_4 x} = \frac{-1}{\log_4 x + 1}$$

$$\frac{3(2+\log_4 x)(1+\log_4 x) + 3\log_4 x(1+\log_4 x)}{\log_4 x(2+\log_4 x)(1+\log_4 x)} = \frac{-1 \log_4 x(2+\log_4 x)}{\log_4 x(2+\log_4 x)(1+\log_4 x)}$$

C.E.
$$\begin{cases} \log_4 x \neq 0 \Rightarrow \log_4 x \neq \log_4 4^0 \Rightarrow \boxed{x \neq 1} \\ 2+\log_4 x \neq 0 \Rightarrow \log_4 x \neq -2 \Rightarrow \log_4 x \neq \log_4 4^{-2} \quad \boxed{x \neq \frac{1}{16}} \\ 1+\log_4 x \neq 0 \Rightarrow \log_4 x \neq -1 \Rightarrow \log_4 x \neq \log_4 4^{-1} \quad \boxed{x \neq \frac{1}{4}} \end{cases}$$

C.E. $x > 0$

$$\boxed{\begin{cases} x \neq 1 \\ x \neq \frac{1}{16} \\ x \neq \frac{1}{4} \\ x > 0 \end{cases}}$$

$$3(2+\log_4 x)(1+\log_4 x) + 3\log_4 x(1+\log_4 x) = -\log_4 x(2+\log_4 x)$$

$$3(2+2\log_4 x + \log_4 x + \log_4^2 x) + 3\log_4 x + 3\log_4^2 x = -2\log_4 x - \log_4^2 x$$

$$6 + 6\log_4 x + 3\log_4 x + 3\log_4^2 x + 3\log_4 x + 3\log_4^2 x = -2\log_4 x - \log_4^2 x$$

$$7\log_4^2 x + 14\log_4 x + 6 = 0 \qquad \log_4 x = t$$

$$7t^2 + 14t + 6 = 0$$

$$\Delta = b^2 - 4ac = (14)^2 - 4(6)(7) = 196 - 168 = 28$$

$$t_1, t_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = -\frac{14 \pm \sqrt{28}}{14}$$

$$\begin{array}{r|l} 28 & 2 \\ 14 & 2 \\ 7 & 7 \end{array}$$

$\sqrt{28} = 2\sqrt{7}$

$$= -\frac{14 \pm 2\sqrt{7}}{14} = \frac{\cancel{2}(-7 \pm \sqrt{7})}{\cancel{14}\,7}$$

$$t_1 = -\frac{7-\sqrt{7}}{7} \quad \vee \quad t_2 = -\frac{7+\sqrt{7}}{7}$$

$$\log_4 x = -\frac{7 \pm \sqrt{7}}{7} \Rightarrow \log_4 x = \log_4 4^{\frac{-7 \pm \sqrt{7}}{7}}$$

$$\boxed{x = 4^{-\frac{7 \pm \sqrt{7}}{7}}}$$

$\simeq 0,148$
$\simeq 0,422$

$$x = 2^{2\left(-\frac{7 \pm \sqrt{7}}{7}\right)} = 2^{-\frac{14 \pm 2\sqrt{7}}{7}}$$

$$2^{-\frac{2\sqrt{7} \pm 2}{\sqrt{7}}}$$

$$-\frac{2\sqrt{7}+2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \frac{-14 + 2\sqrt{7}}{7}$$

RAZIONALIZZANDO

$$\frac{\log_2 x}{\log_2 x+3} - \frac{6}{\log_2 x-3} + \frac{72}{9-\log_2^2 x} = 0$$

$$\frac{\log_2 x}{\log_2 x+3} - \frac{6}{\log_2 x-3} + \frac{72}{(3-\log_2 x)(3+\log_2 x)} = 0$$

$$\frac{\log_2 x}{\log_2 x+3} + \frac{6}{3-\log_2 x} + \frac{72}{(3-\log_2 x)(\log_2 x+3)} = 0$$

$$\frac{\log_2 x(3-\log_2 x)+6(3+\log_2 x)+72}{(3-\log_2 x)(3+\log_2 x)} = 0$$

C.E
$$\begin{cases} 3-\log_2 x \neq 0 \Longrightarrow \log_2 x \neq 3 \Longrightarrow \log_2 x \neq \log_2 2^3 & \boxed{x \neq 8} \\ 3+\log_2 x \neq 0 \Longrightarrow \log_2 x \neq -3 \Longrightarrow \log_2 x \neq \log_2 2^{-3} & x \neq 2^{-3} = \boxed{\frac{1}{8}} \\ x > 0 \end{cases}$$

$$\begin{cases} x \neq 8 \\ x \neq \frac{1}{8} \\ x > 0 \end{cases}$$

$$\log_2 x(3-\log_2 x) + 6(3+\log_2 x) + 72 = 0$$
$$3\log_2 x - \log_2^2 x + 18 + 6\log_2 x + 72 = 0$$
$$-\log_2^2 x + 9\log_2 x + 90 = 0$$
$$\log_2^2 x - 9\log_2 x - 90 = 0 \qquad \log_2 x = t$$
$$t^2 - 9t - 90 = 0$$
$$\Delta = b^2 - 4ac = (-9)^2 - 4(1)(-90) = 81 + 360 = 441$$
$$t_1, t_2 = \frac{-b \pm \sqrt{\Delta}}{2} = \frac{9 \pm \sqrt{441}}{2}$$
$$t_1 = \frac{9-21}{2} = \frac{-12}{2} = -6 \qquad t_2 = \frac{9+21}{2} = \frac{30}{2} = 15$$

$$\boxed{t_1 = -6 \ \wedge \ t_2 = 15}$$

$$\log_2 x = -6 \Rightarrow \log_2 x = \log_2 2^{-6} \qquad x = 2^{-6}$$
$$\log_2 x = 15 \Longrightarrow \log_2 x = \log_2 2^{15} \qquad x = 2^{15}$$

$$\boxed{x_1 = 2^{-6} \ \wedge \ x = 2^{15}}$$

$$\frac{1}{2^6}$$

Disequazioni logaritmiche

$\log_a x \geq 2$

$\quad\quad\quad\leq$

$\log_a [f(x)] \underset{(\leq)}{\geq} \log_a [g(x)]$

se $a > 1 \implies f(x) \underset{(\leq)}{\geq} g(x)$

se $0 < a < 1 \implies f(x) \underset{(\geq)}{\leq} g(x)$

CAMBIA IL VERSO

$-2\ln^2 x + \ln x + 1 > 0$ $\qquad \ln = \log_e$

$$\begin{cases} x > 0 \\ 2\ln^2 x - \ln x - 1 < 0 \end{cases}$$

$\ln x = t$

$2t^2 - t - 1 < 0$

$\Delta = (-1)^2 - 4(2)(-1) = 1 + 8 = 9$

$t_1, t_2 = \frac{1 \pm \sqrt{9}}{4}$

$\frac{1-3}{4} = -\frac{2}{4} = -\frac{1}{2}$

$\frac{1+3}{4} = 1$

$-\frac{1}{2} < t < 1$

$-\frac{1}{2} < \ln x < 1$

$$\begin{cases} \ln x > -\frac{1}{2} \Rightarrow \ln x > \ln e^{-\frac{1}{2}} \\ \ln x < 1 \end{cases}$$

$x > \frac{1}{\sqrt{e}} \cdot \frac{\sqrt{e}}{\sqrt{e}}$

$\boxed{x > \frac{\sqrt{e}}{e}}$

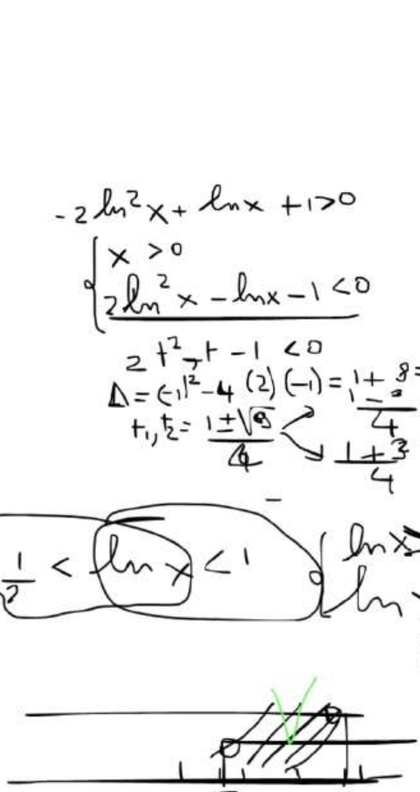
$0{,}606$

$\ln x < \ln e^1$

$\boxed{x < e}$

$2{,}718$

$$\begin{cases} x > \frac{\sqrt{e}}{e} \\ x < e \end{cases}$$



$\frac{\sqrt{e}}{e}$ $\qquad e$

$$\boxed{\frac{\sqrt{e}}{e} < x < e}$$