Lezione 24

$\operatorname{sen} x - \cos x = 0 \qquad \operatorname{sen} x = \cos x \qquad \operatorname{sen} x = \frac{2t}{1+t^2}$

$\frac{2t}{1+t^2} - \frac{(1-t^2)}{1+t^2} = 0 \qquad\qquad \cos x = \frac{1-t^2}{1+t^2}$

$\frac{2t - 1 + t^2}{\cancel{1+t^2}} = \frac{0}{\cancel{1+t^2}} \qquad\qquad t = tg\frac{x}{2} = \frac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}}$

$\cos\frac{x}{2} \neq 0 \qquad \boxed{x \neq \pi + 2k\pi} \qquad \frac{x}{2} \neq \frac{\pi}{2} + k\pi$

$t^2 + 2t - 1 = 0$

$\Delta = 4 - 4(1)(-1) = 4 + 4 = 8$

$t_1, t_2 = -\frac{2 \pm \sqrt{8}}{2} = -\frac{2 \pm 2\sqrt{2}}{2} = \frac{\cancel{2}(-1 \pm \sqrt{2})}{\cancel{2}}$

$t_1 = -1 - \sqrt{2} \qquad\qquad t_2 = -1 + \sqrt{2}$

$tg\frac{x}{2} = -1 - \sqrt{2} \qquad\qquad tg\frac{x}{2} = -1 + \sqrt{2}$

$\frac{x}{2} = arctg(-1 - \sqrt{2}) \qquad\qquad \frac{x}{2} = \frac{\pi}{8} + k\pi$

$\frac{x}{2} = arctg[-(1 + \sqrt{2})] \qquad\qquad x = \cancel{2}\frac{\pi}{\cancel{8}_4} + 2k\pi$

$\pi - \frac{3}{8}\pi$

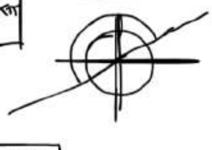
$\frac{8\pi - 3\pi}{8} = \frac{5}{8}\pi$

$\frac{x}{2} = \frac{5}{8}\pi + k\pi \qquad\qquad \boxed{x = \frac{\pi}{4} + 2k\pi}$

$x = 2 \cdot \frac{5}{8}\pi + 2k\pi$

$\boxed{x = \frac{5}{4}\pi + 2k\pi}$

$\frac{5}{\cancel{4}_1} \cdot \cancel{180}^{45} = 225$



$\boxed{x = \frac{\pi}{4} + k\pi}$

$2\,\text{sen}^2x+\sqrt{3}\,\text{sen}x\cos x-\cos^2x-2=0$

$2\,\text{sen}^2x+\sqrt{3}\,\text{sen}x\cos x-\cos^2x-2(\text{sen}^2x+\cos^2x)=0$

$\cancel{2\,\text{sen}^2x}+\sqrt{3}\,\text{sen}x\cos x-\cos^2x-\cancel{2\,\text{sen}^2x}-2\cos^2x=0$

$\sqrt{3}\,\text{sen}x\cos x-3\cos^2x=0$

$\cos x(\sqrt{3}\,\text{sen}x-3\cos x)=0$

$\cos x=0 \Longrightarrow \boxed{x=\frac{\pi}{2}+2k\pi}$

$\sqrt{3}\,\text{sen}x-3\cos x=0$

$\text{sen}x=\frac{2t}{1+t^2}$

$\sqrt{3}\cdot\frac{2t}{1+t^2}-\frac{3(1-t^2)}{1+t^2}=0$

$\cos x=\frac{1-t^2}{1+t^2}$

$\frac{2\sqrt{3}t-3+3t^2}{\cancel{1+t^2}}=\frac{0}{\cancel{1+t^2}}$

$\boxed{t=\text{tg}\frac{x}{2}}$

$\frac{x}{2}\neq\frac{\pi}{2}+k\pi$

$x\neq\pi+2k\pi$

$3t^2+2\sqrt{3}t-3=0$

$\Delta=b^2-4ac=(2\sqrt{3})^2-4(3)(-3)=$

$=12+36=48$

$\begin{array}{r|l} 48 & 2\\ 24 & 2\\ 12 & 2\\ 6 & 2\\ 3 & 3\\ 1 & \end{array}$

$t_1,t_2=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{-2\sqrt{3}\pm\sqrt{48}}{6}$

$\sqrt{2^4\cdot 3}=2^2\sqrt{3}=4\sqrt{3}$

$=\frac{-2\sqrt{3}\pm4\sqrt{3}}{6}$

$\to t_1=-\frac{\cancel{6}\sqrt{3}}{\cancel{6}}=-\sqrt{3}$

$\to t_2=\frac{\cancel{2}\sqrt{3}}{\cancel{6}\,3}=\frac{\sqrt{3}}{3}$

$t_1=-\sqrt{3}$

$\frac{120}{180}$

$t_2=\frac{\sqrt{3}}{3}$

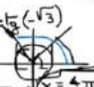
$\text{tg}\frac{x}{2}=-\sqrt{3}$

$\frac{\cancel{12}\,2}{\cancel{18}\,3}$

$\text{tg}\frac{x}{2}=\frac{\sqrt{3}}{3}$

$\frac{x}{2}=\text{arctg}(-\sqrt{3})$

$\frac{x}{2}=\text{arctg}\left(\frac{\sqrt{3}}{3}\right)$



$\frac{x}{2}=\frac{\pi}{6}+k\pi$

$\frac{x}{2}=\frac{2}{3}\pi+k\pi \Rightarrow \boxed{x=\frac{4}{3}\pi+2k\pi}$

$x=\frac{\cancel{2}\,1\pi}{\cancel{6}\,3}+2k\pi$

$\boxed{x=\frac{\pi}{3}+2k\pi}$

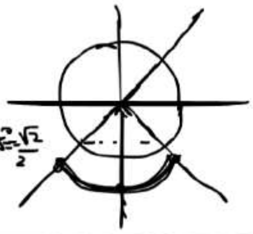
$\boxed{x=\frac{\pi}{3}+k\pi}$

Disequazioni goniometriche

$$\boxed{\operatorname{sen} x > \frac{1}{2}}$$

$$\boxed{\frac{\pi}{6} + 2K\pi < x < \frac{5}{6}\pi + 2K\pi} \quad \boxed{K \in \mathbb{Z}}$$

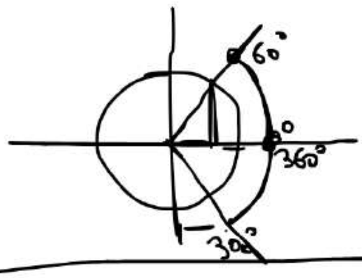
$$\operatorname{sen} x < -\frac{\sqrt{2}}{2}$$



$$\boxed{\frac{5}{4}\pi + 2K\pi < x < \frac{7}{4}\pi + 2K\pi}$$

$$\frac{\cancel{225}\ \cancel{45}\ 5}{\cancel{180}\ \cancel{36}\ 4} \qquad \frac{\cancel{315}\ \cancel{63}\ 7}{\cancel{180}\ \cancel{36}\ 4}$$

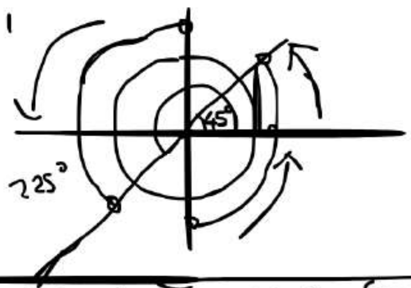
$\cos x > \frac{1}{2}$



300

$$\boxed{0 + 2K\pi \le x < \frac{\pi}{3} + 2K\pi \quad \vee \quad \frac{5\pi}{3} + 2K\pi < x \le 2\pi + 2K\pi}$$

$\operatorname{tg} x < 1$



$\frac{\cancel{225}\ \cancel{45}\ 5}{\cancel{180}\ \cancel{36}\ 4}$

$\underline{K = 1}$

$$\boxed{0 \leq x < \frac{\pi}{4} \vee \frac{\pi}{2} < x < \frac{5}{4}\pi \vee \frac{3}{2}\pi < x \leq 2\pi}$$

$2\,sen^2x + 3\,sen\,x + 1 < 0$ $\qquad sen\,x = t$

$2t^2 + 3t + 1 < 0$

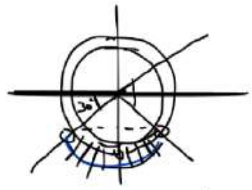
$\Delta = b^2 - 4ac = 3^2 - 4(2)(1) = 9 - 8 = 1$

$t_1, t_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{-3 \pm 1}{4}$ $\to -\frac{\cancel{2}^{1}}{\cancel{4}_{2}} = -\frac{1}{2}$; $\to -\frac{\cancel{4}^{1}}{\cancel{4}_{1}} = -1$

$t_1 = -1 \lor t_2 = -\frac{1}{2}$

$\boxed{-1 < t < -\frac{1}{2}}$

$\boxed{-1 < sen\,x < -\frac{1}{2}} \to \begin{cases} sen\,x > -1 \\ sen\,x < -\frac{1}{2} \end{cases}$



$\frac{210°}{180°} = \frac{\cancel{21}^{7}}{\cancel{18}_{6}}$

$\frac{330°}{180°} = \frac{\cancel{33}^{11}}{\cancel{18}_{6}}$

$\frac{7}{6}\pi + 2k\pi < x < \frac{3}{2}\pi + 2k\pi$

$\lor$

$\frac{3}{2}\pi + 2k\pi < x < \frac{11}{6}\pi + 2k\pi$

$\boxed{\frac{7}{6}\pi + 2k\pi < x < \frac{11}{6}\pi + 2k\pi \ \land \ x \neq \frac{3}{2}\pi + 2k\pi \quad k \in \mathbb{Z}}$

$\operatorname{sen} x - \sqrt{3}\cos x > \sqrt{3}$

$\operatorname{sen} x = \dfrac{2t}{1+t^2}$

$\cos x = \dfrac{1-t^2}{1+t^2}$

$t = \operatorname{tg}\dfrac{x}{2}$

$x \neq \pi + 2k\pi$

$\dfrac{2t}{1+t^2} - \sqrt{3}\cdot\dfrac{(1-t^2)}{1+t^2} > \sqrt{3}$

$\dfrac{2t - \sqrt{3} + \sqrt{3}t^2 - \sqrt{3}(1+t^2)}{1+t^2} > 0$

$\dfrac{2t - \sqrt{3} + \cancel{\sqrt{3}t^2} - \sqrt{3} - \cancel{\sqrt{3}t^2}}{1+t^2} > 0$

$\dfrac{2t - 2\sqrt{3}}{1+t^2} > 0$

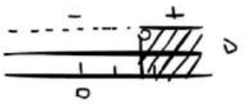
N $\quad 2t - 2\sqrt{3} > 0$

$t - \sqrt{3} > 0$

$t > \sqrt{3}$

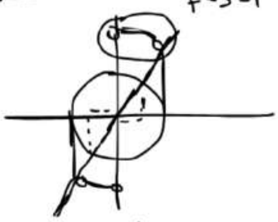
D $\quad 1 + t^2 > 0$

$t^2 > -1 \quad \forall t \in \mathbb{R}$



$t > \sqrt{3}$

$\operatorname{tg}\dfrac{x}{2} > \sqrt{3}$



$\dfrac{\pi}{3} + k\pi < \dfrac{x}{2} < \dfrac{\pi}{2} + k\pi$

$\boxed{2\dfrac{\pi}{3} + 2k\pi < x < \pi + 2k\pi} \quad \boxed{k \in \mathbb{Z}}$