Lezione 26

$$y = f(x) = \frac{x^2 - 8x + 12}{x^2 - 6x + 5}$$

$$f'(x) = \frac{(2x-8)(x^2-6x+5) - (x^2-8x+12)\cdot(2x-6)}{(x^2-6x+5)^2}$$

$$f'(x) = \frac{2x^3 - 12x^2 + 10x - 8x^2 + 48x - 40 - (2x^3 - 6x^2 - 16x^2 + 48x + 24x - 72)}{(x^2-6x+5)^2} =$$

$$f'(x) = \frac{\cancel{2x^3} - 20x^2 + 58x - 40 - (\cancel{2x^3} - 22x^2 + 72x - 72)}{(x^2-6x+5)^2}$$

$$\boxed{f'(x) = \frac{2x^2 - 14x + 32}{(x^2-6x+5)^2}}$$

A $(2;0)$
B $(6;0)$

$$f'(x_A) = f'(2) = \frac{8 - 28 + 32}{(4-12+5)^2} = \frac{-20+32}{(-3)^2} = \boxed{\frac{\cancel{12}\,4}{\cancel{9}\,3}} = \frac{4}{3}$$

$$\boxed{m_1 = \frac{4}{3}}$$

A $(2;0)$

$$y - y_A = m(x - x_A)$$

$$y = \frac{4}{3}(x-2) \Rightarrow \boxed{y = \frac{4}{3}x - \frac{8}{3}}$$

$$f'(x_B) = f'(6) = \frac{2\cdot 36 - 14\cdot 6 + 32}{(\cancel{36} - \cancel{36} + 5)^2} = \frac{72 - 84 + 32}{25} = \frac{\cancel{20}\,4}{\cancel{25}\,5} = \boxed{\frac{4}{5}}$$

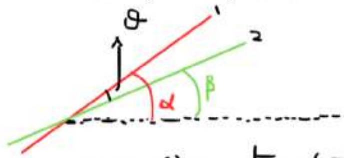
B $(6;0)$

$$m_2 = \frac{4}{5}$$

$$y - y_B = m_2(x - x_B)$$

$$y = \frac{4}{5}(x-6) = \frac{4}{5}x - \frac{24}{5} \Rightarrow \boxed{y = \frac{4}{5}x - \frac{24}{x}}$$

Ricaviamo l'angolo formato l'angolo fra le due rette

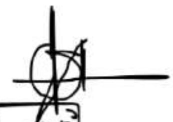


$$\vartheta = \alpha - \beta$$

$$\boxed{\begin{aligned} m_1 &= tg\,\alpha \\ m_2 &= tg\,\beta \end{aligned}}$$

$$tg\,\vartheta = tg(\alpha - \beta) = \frac{tg\,\alpha - tg\,\beta}{1 + tg\,\alpha\, tg\,\beta}$$

$$\boxed{tg\,\vartheta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \vartheta < 90^\circ}$$



$$\boxed{\vartheta < 90^\circ}$$

$$m_1 = \frac{4}{3} \qquad m_2 = \frac{4}{5}$$

$$tg\,\vartheta = \left| \frac{\frac{4}{3} - \frac{4}{5}}{1 + \frac{4}{3} \cdot \frac{4}{5}} \right|$$

$$tg\,\vartheta = \left| \frac{\frac{20 - 12}{15}}{1 + \frac{16}{15}} \right| \Rightarrow tg\,\vartheta = \left| \frac{\frac{8}{15}}{\frac{15+16}{15}} \right|$$

$$tg\,\vartheta = \left| \frac{8}{\cancel{15}} \cdot \frac{\cancel{15}}{31} \right| \qquad tg\,\vartheta = \frac{8}{31}$$

$$\boxed{\vartheta = arctg\,\frac{8}{31}}$$

$$y = \frac{x^2+2}{x(x+1)} = \frac{x^2+2}{x^2+x}$$

Dom f: $x(x+1) \neq 0$ $\quad x \neq 0$, $x \neq -1$

$\mathbb{R} \setminus \{0, -1\}$

$$f(-x) = \frac{x^2+2}{-x(-x+1)} = \frac{x^2+2}{x^2-x} = \frac{x^2+2}{x(x-1)} \neq \begin{cases} -f(x) \\ f(x) \end{cases}$$

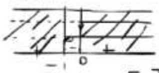
NON HA PARITÀ

$$\begin{cases} y = \frac{x^2+2}{x(x+1)} \\ y = 0 \end{cases} \Rightarrow \begin{array}{l} x^2+2=0 \\ x^2 = -2 \end{array} \quad \nexists x \in \text{Dom} f$$

$$\frac{x^2+2}{x(x+1)} \geq 0$$

N $x^2+2 \geq 0$ $\forall x \in \text{Dom} f$

D_1 $x > 0$ $\quad D_2$ $x > -1$



$I_P =]-\infty; -1[\cup]0; +\infty[$

$I_N =]-1; 0[$

$\lim_{x \to \infty} \frac{1x^2+2}{1x^2+x} = 1 \Rightarrow \boxed{y=1}$ ASINTOTO ORIZZONTALE

$\lim_{x \to 0^+} \frac{x^2+2}{x^2+x} = \frac{0+2}{0+0} = \frac{2}{0} = +\infty$

$\lim_{x \to 0^-} \frac{x^2+2}{x^2+x} = -\infty$ $\quad \boxed{x=0}$ ASINTOTO VERTICALE

$\lim_{x \to -1} \frac{x^2+2}{x^2+x} = \frac{1+2}{1-1} = \frac{3}{0} = -\infty$

$\lim_{x \to -1} \frac{x^2+2}{x^2+x} =$ $\quad \boxed{x=-1}$ ASINTOTO VERTICALE

$$f'(x) = \frac{2x(x^2+x) - (x^2+2)(2x+1)}{(x^2+x)^2}$$

$$f'(x) = \frac{2x^3+2x^2-(2x^3+x^2+4x+2)}{(x^2+x)^2}$$

$$f'(x) = \frac{\cancel{2x^3}+2x^2-\cancel{2x^3}-x^2-4x-2}{(x^2+x)^2}$$

$$f'(x) = \frac{x^2-4x-2}{(x^2+x)^2}$$

$f'(x)=0$ $\quad x^2-4x-2=0$

$\Delta = 16-4(1)(-2) = 16+8 = 24$

$\begin{array}{r|l} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}$

$x_1, x_2 = \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} =$

$\sqrt{24} = 2\sqrt{6}$ $\quad = \frac{\cancel{2}(2 \pm \sqrt{6})}{\cancel{2}}$

$x = 2-\sqrt{6} \quad \vee \quad x = 2+\sqrt{6}$ $\qquad \frac{x^2+2}{x^2+x}$

$f(2-\sqrt{6}) = \frac{(2-\sqrt{6})^2+2}{(2-\sqrt{6})^2+2-\sqrt{6}} = \frac{4+6-4\sqrt{6}+2}{4+6-4\sqrt{6}+2-\sqrt{6}} =$

$= \frac{12-4\sqrt{6}}{12-5\sqrt{6}} \cdot \frac{12+5\sqrt{6}}{12+5\sqrt{6}} = \frac{144+60\sqrt{6}-48\sqrt{6}-120}{144-150} =$

$= \frac{24+12\sqrt{6}}{-6} = \frac{\cancel{6}(4+2\sqrt{6})}{-\cancel{6}} = -4-2\sqrt{6}$

$\boxed{A(2-\sqrt{6}\,;\,-4-2\sqrt{6})}$ $\quad \boxed{B(2+\sqrt{6}\,;\,-4+2\sqrt{6})}$

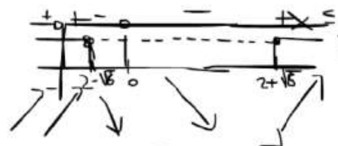
$f'(x) \geq 0 \qquad \frac{x^2-4x-2}{(x^2+x)^2} \geq 0$

$N \quad x^2-4x-2 \geq 0$

$x \leq 2-\sqrt{6} \vee x \geq 2+\sqrt{6}$

$D \quad (x^2+x)^2 > 0$

$\forall x \in Dom f$



$A\,(2-\sqrt{6}\,;\,-4-2\sqrt{6})$ MASSIMO RELATIVO

$B\,(2+\sqrt{6}\,;\,-4+2\sqrt{6})$ MINIMO RELATIVO

$$f''(x) = \frac{(2x-4)\cdot(x^2+x)^2 - (x^2-4x-2)\,2(x^2+x)(2x+1)}{(x^2+x)^4}$$

$$= \frac{\cancel{(x^2+x)}\left[(2x-4)(x^2+x) - 2(x^2-4x-2)(2x+1)\right]}{(x^2+x)^{\cancel{4}}} =$$

$$= \frac{2x^3+2x^2-4x^2-4x-2(2x^3+x^2-8x^2-4x-4x-2)}{(x^2+x)^3}$$

$$= \frac{2x^3+\cancel{2x^2}-4x^2-4x-4x^3-\cancel{2x^2}+16x^2+8x+8x+4}{(x^2+x)^3}$$

$$= \frac{-2x^3+12x^2+12x+4}{(x^2+x)^3}$$

$$= \frac{-2(x^3-6x^2-6x-2)}{(x^2+x)^3}$$

$f'(x)=0$ $\quad x^3-6x^2-6x-2=0$

$f''(x) \geq 0$