Lezione 18

Funzioni continue

$$\boxed{\lim_{x \to c} f(x) = f(c)}$$

1) $\exists\ f(c)$

2) $\exists \lim_{x \to c} f(x) = f(c)$

3) $\lim_{x \to c} f(x) = f(c)$

$y = x^2$ È CONTINUA IN $\boxed{x = c = 4}$

1) $f(4) = 4^2 = 16$ ✓

2) $\lim_{x \to 4} x^2 = 4^2 = 16$ ✓

3) $\lim_{x \to 4} x^2 = f(4) = 16$ ✓

$\Longrightarrow$ $\boxed{f \text{ È CONTINUA IN } x = 4}$

$f(x) = 2^x$ È CONTINUA IN $x = c = 4$

1) $f(4) = 2^4 = 16$ ✓

2) $\lim_{x \to 4} 2^x = 2^4 = 16$ ✓

3) $\lim_{x \to 4} 2^x = f(4) = 2^4 = 16$ ✓

$f(x) = 2^x$ CONTINUA IN $x = 4$

$$y = f(x) = \log_2(x) \qquad \boxed{x > 0}$$

$$\mathrm{Dom}\, f = \mathbb{R}^+$$

FUNZIONI RAZIONALI FRATTE RIPRESA CALCOLO LIMITI

$$\lim_{x\to+\infty} \frac{2x^3+1}{-3x^2+1} = \lim_{x\to+\infty} \frac{x^3\left(2+\frac{1}{x^3}\right)}{x^2\left(-3+\frac{1}{x^2}\right)} = \lim_{x\to+\infty} \frac{x\left(2+\frac{1}{x^3}\right)}{\left(-3+\frac{1}{x^2}\right)}$$

$$= -\frac{2}{3}\cdot(+\infty) = \boxed{-\infty}$$

$$\lim_{x\to-\infty} \frac{x^5-x^3+x}{x^3+1} = \lim_{x\to-\infty} \frac{\cancel{x^5}^{\,x^2}\left(1-\frac{1}{x^2}+\frac{1}{x^4}\right)}{\cancel{x^3}\left(1+\frac{1}{x^3}\right)} = (-\infty)^2\cdot 1 = \boxed{+\infty}$$

$$\lim_{x\to-\infty} \frac{x-x^2}{4+3x-x^4} = \lim_{x\to-\infty} \frac{\cancel{x^2}\left(-1+\frac{1}{x}\right)}{\cancel{x^4}_{\,x^2}\left(-1+\frac{3}{x^3}+\frac{4}{x^4}\right)} = \frac{1}{(-\infty)^2}\cdot 1$$

$$0\cdot 1 = 0$$

$$\lim_{x\to\pm\infty} \frac{5x^3-3x^2+2x-1}{8x^3+3} =$$

$$= \lim_{x\to\pm\infty} \frac{\cancel{x^3}\left(5-\frac{3}{x}+\frac{2}{x^2}-\frac{1}{x^3}\right)}{\cancel{x^3}\left(8+\frac{3}{x^3}\right)} = \frac{5}{8}$$

a) grado num > grado den $\Longrightarrow \infty$

b) grado num < grado den $\Longrightarrow 0$

c) grado num = grado den $\Longrightarrow l$ $\quad l \in \mathbb{R}$

$$\lim_{x \to 3} \frac{x^3 - 3x^2 + x - 3}{x^2 - 2x - 3} = \frac{27 - 3 \cdot 3^2 + 3 - 3}{3^2 - 2 \cdot 3 - 3} =$$

$$= \frac{\cancel{27} - \cancel{27} + \cancel{3} - \cancel{3}}{\cancel{9} - \cancel{6} - \cancel{3}} = \boxed{\left[\frac{0}{0}\right]}$$

FORMA INDETERMINATA

$$= \underbrace{x^3 - 3x^2} + \underbrace{x - 3} = x^2(x-3) + 1(x-3) = \boxed{(x-3)(x^2+1)}$$

$$x^2 - 2x - 3 =$$

$p = -3$ $\quad$ $1 \cdot 3$

$s = -2$ $\quad$ $a = 1$ $\quad$ $b = -3$

$$= (x+a)(x+b)$$

$$= (x+1)(x-3)$$

$$\lim_{x \to 3} \frac{\cancel{(x-3)}(x^2+1)}{(x+1)\cancel{(x-3)}} = \lim_{x \to 3} \frac{x^2+1}{x+1} = \frac{3^2+1}{3+1} = \frac{\cancel{10}^{5}}{\cancel{4}_{2}}$$

$$= \frac{5}{2}$$

Limiti notevoli

1°

$$\boxed{\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x} = e}$$

$e \simeq 2,718\ldots\ldots$

$x = 1 \qquad \left(1 + \frac{1}{1}\right)^{1} = (1+1)^{1} = 2^{1} = 2$

$x = 10 \qquad \left(1 + \frac{1}{10}\right)^{10} = (1,1)^{10} \simeq 2,593$

$x = 100 \qquad \left(1 + \frac{1}{100}\right)^{100} = (1 + 0,01)^{100} = (1,01)^{100} \simeq 2,704$

$x = 1000 \qquad \left(1 + \frac{1}{1000}\right)^{1000} = (1 + 0,001)^{1000} = (1,001)^{1000} \simeq 2,716\ldots$

$$\boxed{\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{x}}$$

$\boxed{\frac{1}{x} = z}$ CAMBIAM. DI VARIABILI

se $x \to \infty \Rightarrow z \to 0$

$x = \frac{1}{z}$

VALORE DEL LIMITE NON CAMBIA!!!

$$\lim_{z \to 0} (1 + z)^{\frac{1}{z}} = e$$

2°

$$\boxed{\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e}$$

$$\lim_{x \to \infty} \left(1 - \frac{2}{x}\right)^{x}$$

$$\boxed{-\frac{2}{x} = \frac{1}{z}}$$

$$\lim_{z \to \infty} \left(1 + \frac{1}{z}\right)^{-2z} =$$

$$x \to \infty \Longrightarrow z \to \infty$$

$$\lim_{z \to \infty} \left[\left(1 + \frac{1}{z}\right)^{z}\right]^{-2}$$

$$-\frac{2}{x} = \frac{1}{z}$$

$$\boxed{-2z = x}$$

$$\Big[\underbrace{\lim_{z \to \infty} \left(1 + \frac{1}{z}\right)^{z}}\Big]^{-2}$$

$$= e^{-2} = \frac{1}{e^{2}}$$

$$\frac{\log 1}{0} = \left[\frac{0}{0}\right]$$

$a \in \mathbb{R}^+$

$a > 1$

$0 < a < 1$

$$\lim_{x \to 0} \frac{\log_a (1+x)}{x} =$$

$$= \lim_{x \to 0} \frac{1}{x} \log_a (1+x)$$

$$m \log_a b = \log_a b^m$$

$$= \lim_{x \to 0} \log_a (1+x)^{\frac{1}{x}} = \log_a \boxed{\lim_{x \to 0} (1+x)^{\frac{1}{x}}} \to e$$

$$= \log_a e$$

$$\boxed{\begin{aligned} &\lim_{x \to 0} \frac{\log_a (1+x)}{x} = \log_a e \\ &\lim_{x \to 0} \frac{\log_e (1+x)}{x} = \log_e e = 1 \end{aligned}}$$

$$\boxed{e^{x} = e^{1} = e}$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} =$$

$$= \lim_{t \to 0} \frac{t}{\log_a(t+1)} = \frac{1}{\log_a e} = \log_e a$$

$a^x - 1 = t$

$a^x = t + 1$

$x = \log_a(t+1)$

$x \to 0$ $\quad 1 - 1 = 0 = t$

$t \to 0$

$$\boxed{\begin{aligned} &\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a \\ &\lim_{x \to 0} \frac{e^x - 1}{x} = \log_e e = 1 \end{aligned}}$$