Lezione 19

$$\lim_{x \to 1} \frac{x-1}{x^3-4x^2-4x-1} = \frac{1-1}{1-4+4-1} = \left[\frac{0}{0}\right]$$

x^3-4x^2+4x-1 $\quad D_1 = \{\pm 1\}$

$P(1) = 1-4+4-1 = 0$ $\quad (x-1)$

$$\begin{array}{c|ccc|c} & 1 & -4 & 4 & -1 \\ 1 & & 1 & -3 & 1 \\ \hline & 1 & -3 & 1 & 0 \end{array}$$

$$x^3-4x^2+4x-1 = \boxed{(x-1)(x^2-3x+1)}$$

$P=1$
$S=-3$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x^2-3x+1)} =$$

$$= \lim_{x \to 1} \frac{1}{x^2-3x+1} = \frac{1}{1-3+1} = \frac{1}{-1} = -1$$

$(\sqrt{3})^2$

$$\lim_{x \to \sqrt{3}} \frac{3-x^2}{\sqrt{3}-x} = \frac{3-3}{\sqrt{3}-\sqrt{3}} = \left[\frac{0}{0}\right]$$

$$\lim_{x \to \sqrt{3}} \frac{\cancel{(\sqrt{3}-x)}(\sqrt{3}+x)}{\cancel{(\sqrt{3}-x)}}$$

$3-x^2 = (\sqrt{3}-x)(\sqrt{3}+x)$

$a^2-b^2 = (a-b)(a+b)$

$$\lim_{x \to \sqrt{3}} \sqrt{3}+x = \sqrt{3}+\sqrt{3} = 2\sqrt{3}$$

$$\lim_{x \to 2^+} \frac{x^2-4}{x^2-4x+4} = \frac{2^2-4}{2^2-8+4} = \frac{4-4}{4-8+4} = \left[\frac{0}{0}\right]$$

$$\lim_{x \to 2^+} \frac{\cancel{(x-2)}(x+2)}{(x-2)^{\cancel{2}}} = \lim_{x \to 2^+} \frac{(x+2)}{x-2} = \frac{2+2}{2-2}$$

$$= \frac{4}{0} = +\infty$$

$$\lim_{x \to \sqrt{2}} \frac{x^2-2}{2\sqrt{2}-x^3} = \frac{(\sqrt{2})^2-2}{2\sqrt{2}-(\sqrt{2})^3} = \frac{2-2}{2\sqrt{2}-2\sqrt{2}} = \left[\frac{0}{0}\right]$$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

$$\lim_{x \to \sqrt{2}} \frac{(x-\sqrt{2})(x+\sqrt{2})}{(\sqrt{8}-x)(\sqrt{8^2}+\sqrt{8}x+x^2)}$$

$x^2-2 = (x-\sqrt{2})(x+\sqrt{2})$

$\sqrt[6]{8} = \sqrt[6]{2^3} = \sqrt{2}$

$2\sqrt{2}-x^3 =$
$= \sqrt{8}-x^3 = (\sqrt[6]{8}-x)(\sqrt[6]{8^2}+\sqrt[6]{8}x+x^2)$

$\sqrt[3]{\sqrt{8}} = \sqrt[6]{8}$

$$\lim_{x \to \sqrt{2}} \frac{(x-\sqrt{2})(x+\sqrt{2})}{(\sqrt[6]{8}-x)(\sqrt[6]{8^2}+\sqrt[6]{8}x+x^2)} =$$

$$= \lim_{x \to \sqrt{2}} \frac{\cancel{(x-\sqrt{2})}(x+\sqrt{2})}{-\cancel{(x-\sqrt{2})}(\sqrt[6]{8^2}+\sqrt[6]{8}x+x^2)} = -\frac{\sqrt{2}+\sqrt{2}}{\sqrt[6]{64}+\sqrt[6]{8}\cdot\sqrt{2}+2} =$$

$$= -\frac{2\sqrt{2}}{\sqrt[6]{2^6}+2\sqrt{2}\sqrt{2}+2} = -\frac{2\sqrt{2}}{2+2\cdot 2+2} = -\frac{2\sqrt{2}}{2+4+2} =$$

$$= -\frac{\cancel{2}\sqrt{2}}{\cancel{8}\,4} = -\frac{\sqrt{2}}{4}$$

$$\lim_{x \to 2} \frac{e^x - e^2}{x - 2} =$$

$$\boxed{\lim_{x \to 0} \frac{e^{x} - 1}{x} = 1}$$

$$= \lim_{x \to 2} \frac{e^2\left(\frac{e^x}{e^2} - 1\right)}{x - 2} = e^2 \lim_{x \to 2} \frac{e^{x-2} - 1}{x-2}$$

$x - 2 = t$

$x \to 2 \Rightarrow t \to 0$

$$e^2 \lim_{t \to 0} \frac{e^t - 1}{t} = 1 \cdot e^2 = e^2$$

$$\overset{H}{=} \lim_{x \to 2} \frac{e^x}{1} = e^2$$

$$\lim_{x \to 0} \frac{x}{1 - e^{3x}} = \frac{0}{1-1} = \left[\frac{0}{0}\right]$$

$$\lim_{x \to 0} \frac{x}{1 - e^{3x}} = \lim_{x \to 0} \frac{x}{-(e^{3x} - 1)} =$$

$$= -1 \cdot \lim_{x \to 0} \frac{3x}{e^{3x} - 1} \cdot \frac{1}{3} = \to 1$$

$$= -\frac{1}{3} \boxed{\lim_{x \to 0} \frac{3x}{e^{3x} - 1}} = \boxed{-\frac{1}{3}}$$

$$\boxed{\overset{H}{=} \lim_{x \to 0} \frac{1}{-3e^{3x}} = \boxed{-\frac{1}{3}}}$$

$$\lim_{x \to 0} \frac{1 - e^{4x}}{7x} = \lim_{x \to 0} \frac{-(e^{4x} - 1)}{7x} =$$

$$= -1 \cdot \lim_{x \to 0} \frac{e^{4x} - 1}{\cancel{7x} \frac{4}{7}} \cdot \frac{4}{7} = \to 1 \qquad 7 \cdot \frac{4}{7}$$

$$= -1 \cdot \frac{4}{7} \lim_{x \to 0} \frac{e^{4x} - 1}{4x} = -1 \cdot \frac{4}{7} = \boxed{-\frac{4}{7}}$$

$$\overset{H}{=} \lim_{x \to 0} \frac{1 - e^{4x}}{7x} \overset{H}{=} \lim_{x \to 0} \frac{-4e^{4x}}{7} = \boxed{-\frac{4}{7}}$$

$$\lim_{x \to 0} \frac{e^{3x} - 1}{5x} = \lim_{x \to 0} \frac{e^{3x} - 1}{\cancel{5}x \cdot \frac{3}{\cancel{5}}} \cdot \frac{3}{5} =$$

$$= \frac{3}{5} \lim_{x \to 0} \frac{e^{3x} - 1}{3x} \to 1 = \boxed{\frac{3}{5}}$$

$$\overset{H}{=} \boxed{\lim_{x \to 0} \frac{3e^{3x}}{5} = \boxed{\frac{3}{5}}}$$

Altri limiti notevoli

$K \in \mathbb{R}$

$$\boxed{\lim_{x \to 0} \frac{(1+x)^{K} - 1}{x} = K}$$

Cambiamento di variabile

$(1+x)^K - 1 = z$

$x \to 0 \quad z \to 0$

$e^K = a$

$(1+x)^K = z+1$

$\log(1+x)^K = \log(z+1)$

$K \log(1+x) = \log(1+z)$

$$\lim_{x \to 0} \frac{(1+x)^K - 1}{x} = \lim_{\substack{z \to 0 \\ x \to 0}} \frac{z}{x} = \lim_{\substack{x \to 0 \\ z \to 0}} \frac{K \log(1+x)}{\log(1+z)} \cdot \frac{z}{x} =$$

$$= \lim_{x \to 0} \frac{K \log(1+x)}{x} \cdot \lim_{z \to 0} \frac{z}{\log(1+z)} \to 1$$

$$\boxed{\lim_{x \to 0} \frac{\log_e(1+x)}{x}}$$

$= K \cdot 1 \cdot 1 = K.$

C.V.D

$$\lim_{x \to 0} \frac{\sqrt[3]{(1+x)} - 1}{x} = \lim_{x \to 0} \frac{(1+x)^{\frac{1}{3}} - 1}{x} = \frac{1}{3}$$

$(1+x)^{\frac{1}{3}}$

$$\lim_{x \to 0} \frac{\sqrt[5]{1+x} - 1}{e^x - 1} = \lim_{x \to 0} \frac{(1+x)^{\frac{1}{5}} - 1}{x} \cdot \frac{x}{e^x - 1}$$

$$= \lim_{x \to 0} \frac{(1+x)^{\frac{1}{5}} - 1}{x} \cdot \lim_{x \to 0} \frac{x}{e^x - 1} \to 1 \qquad \lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

$$= \frac{1}{5} \cdot 1$$

$(1+x)^{\frac{1}{5}}$

$\frac{1-5}{5}$

$$\overset{H}{=} \lim_{x \to 0} \frac{\sqrt[5]{1+x} - 1}{e^x - 1} \overset{H}{=} \lim_{x \to 0} \frac{\frac{1}{5}(1+x)^{\frac{1}{5}-1}}{e^x} \to 1$$

$$= \lim_{x \to 0} \frac{\frac{1}{5}(1+x)^{-\frac{4}{5}}}{e^x} = \frac{1}{5} \lim_{x \to 0} \frac{(1+x)^{-\frac{4}{5}}}{e^x} = \boxed{\frac{1}{5}}$$

$[f(x)]^{\alpha}$

$$\frac{d\,[f(x)]^{\alpha}}{dx} = \alpha\,[f(x)]^{\alpha - 1} \qquad \alpha \in \mathbb{R}$$