Lezione 23

$c = \frac{1}{7}$

$a^2 = b^2 - \frac{1}{49}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$F\left(0; -\frac{1}{7}\right)$

No! $b^2 = a^2 - c^2$

Sì! $a^2 = b^2 - c^2$

$\begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \\ x = 0 \end{cases}$

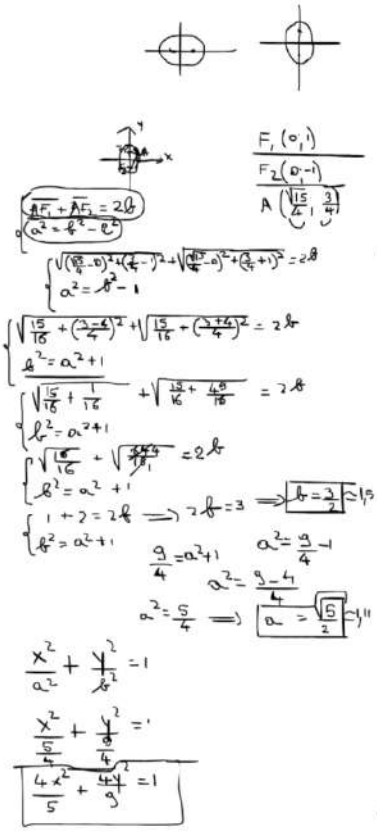
$\frac{y^2}{b^2} = 1 \Longrightarrow y^2 = b^2$

$y = \pm b$

$V_1 = (-b; 0)$

$V_2 = (b; 0)$

IMPOSS.



$F_1(0,1)$

$F_2(0,-1)$

$A\left(\frac{\sqrt{15}}{4}, \frac{3}{4}\right)$

$\overline{AF_1} + \overline{AF_2} = 2b$

$a^2 = b^2 - c^2$

$\begin{cases} \sqrt{\left(\frac{\sqrt{15}}{4}-0\right)^2+\left(\frac{3}{4}-1\right)^2}+\sqrt{\left(\frac{\sqrt{15}}{4}-0\right)^2+\left(\frac{3}{4}+1\right)^2} = 2b \\ a^2 = b^2 - 1 \end{cases}$

$\begin{cases} \sqrt{\frac{15}{16}+\left(\frac{3-4}{4}\right)^2}+\sqrt{\frac{15}{16}+\left(\frac{3+4}{4}\right)^2} = 2b \\ b^2 = a^2+1 \end{cases}$

$\begin{cases} \sqrt{\frac{15}{16}+\frac{1}{16}}+\sqrt{\frac{15}{16}+\frac{49}{16}} = 2b \\ b^2 = a^2+1 \end{cases}$

$\begin{cases} \sqrt{\frac{64}{16}}+\sqrt{\frac{64}{16}} = 2b \\ b^2 = a^2+1 \end{cases}$

$\begin{cases} 1+3 = 2b \Rightarrow 2b = 3 \Rightarrow \boxed{b = \frac{3}{2}} \simeq 1{,}5 \\ b^2 = a^2+1 \end{cases}$

$\frac{9}{4} = a^2+1 \qquad a^2 = \frac{9}{4} - 1$

$a^2 = \frac{9-4}{4}$

$a^2 = \frac{5}{4} \Rightarrow \boxed{a = \frac{\sqrt{5}}{2}} \simeq 1{,}11$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x^2}{\frac{5}{4}} + \frac{y^2}{\frac{9}{4}} = 1$

$\boxed{\frac{4x^2}{5} + \frac{4y^2}{9} = 1}$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$A(0;1)$ $\quad \frac{1}{b^2}=1 \rightarrow b^2=1$ $\quad \boxed{b=1}$

$A(0;1)$

$y=-\frac{\sqrt{3}}{2}x+\frac{\sqrt{5}}{2}$

RETTA TANGENTE

$$\begin{cases} \frac{x^2}{a^2}+\frac{y^2}{1}=1 \\ y=-\frac{\sqrt{3}}{2}x+\frac{\sqrt{5}}{2} \end{cases}$$

$$\frac{x^2}{a^2}+\left(-\frac{\sqrt{3}}{2}x+\frac{\sqrt{5}}{2}\right)^2=1 \qquad \boxed{\Delta=0}$$

$$\frac{x^2}{a^2}+\frac{3}{4}x^2+\frac{5}{4}-\frac{\sqrt{15}}{2}x=1$$

$$\frac{x^2}{a^2}+\frac{3}{4}x^2-\frac{\sqrt{15}}{2}x+\frac{5}{4}-1=0$$

$$\frac{4x^2+3a^2x^2-2\sqrt{15}a^2x+5a^2-4a^2}{4a^2}=0$$

$$(4+3a^2)x^2-2\sqrt{15}a^2x+a^2=0$$

$$\Delta=(2\sqrt{15}a^2)^2-4(4+3a^2)\cdot a^2=0$$

$$4\cdot 15\,a^4-16a^2-12a^4=0 \qquad \Delta=0$$

$$48a^4-16a^2=0$$

$$3a^4-a^2=0$$

$$a^2(3a^2-1)=0 \qquad \boxed{a^2=\frac{1}{3}}$$

$$\frac{x^2}{\frac{1}{3}}+\frac{y^2}{1}=1$$

$$\boxed{3x^2+y^2=1}$$

$$mx^2 + ny^2 + px + qy + r = 0$$

$$\frac{x^2}{5} + \frac{5}{12}y^2 + \frac{12}{5}x - \frac{5\sqrt{3}}{3}y + \frac{56}{5} = 0$$

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

$$\frac{12x^2 + 25y^2 + 144x - 100\sqrt{3}y + 672}{60} = \frac{0}{60}$$

$$12x^2 + 25y^2 + 144x - 100\sqrt{3}y + 672 = 0$$

$$12x^2 + 144x + 25y^2 - 100\sqrt{3}y + 672 = 0$$

$$12(x^2 + 12x) + 25(y^2 - 4\sqrt{3}y) + 672 = 0$$

$$12\left[(x^2 + 12x + 36) - 36\right] + 25\left[(y^2 - 4\sqrt{3}y + 12) - 12\right] + 672 = 0$$

$$12\left[(x+6)^2 - 36\right] + 25\left[(y - 2\sqrt{3})^2 - 12\right] + 672 = 0$$

$$12(x+6)^2 - 432 + 25(y-2\sqrt{3})^2 - 300 + 672 = 0$$

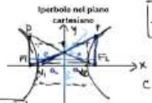
$$\frac{12(x+6)^2}{60} + \frac{25(y-2\sqrt{3})^2}{60} - \frac{60}{60} = 0$$

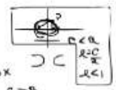
$$\frac{1}{5}(x+6)^2 + \frac{5}{12}(y-2\sqrt{3})^2 = 1$$

$$C(-6, 2\sqrt{3})$$

$$\frac{x^2}{5} + \frac{5}{12}y^2 = 1$$

Iperbole nel piano cartesiano





$c > a$

$F_1 = (-c, 0)$
$F_2 = (c, 0)$ FUOCHI

$V_1 = (-a, 0)$
$V_2 = (a, 0)$ VERTICI

SONO 2

ASSE X FOCALE
ASSE Y (TRASVERSO)!!!

$PF_1 - PF_2 = 2a$ IMPRECISA

$|PF_1 - PF_2| = 2a$

$PF_1 - PF_2 = \pm 2a$

$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = \pm 2a$

$\sqrt{(x+c)^2 + y^2} = \pm 2a + \sqrt{(x-c)^2 + y^2}$

ELEVO AL QUADRATO

$(x+c)^2 + y^2 = 4a^2 + (x-c)^2 + y^2 \pm 4a\sqrt{(x-c)^2 + y^2}$

$x^2 + c^2 + 2cx + y^2 = 4a^2 + x^2 + c^2 - 2cx + y^2 \pm 4a\sqrt{(x-c)^2 + y^2}$

$4cx - 4a^2 = \pm 4a\sqrt{(x-c)^2 + y^2}$

$-a^2 + cx = \pm a\sqrt{(x-c)^2 + y^2}$

$a^4 + c^2x^2 - 2a^2cx = a^2[x^2 + c^2 - 2cx + y^2]$

$a^4 + c^2x^2 - 2a^2cx = a^2x^2 + a^2c^2 - 2a^2cx + a^2y^2$

$a^4 + c^2x^2 - a^2x^2 - a^2c^2 - a^2y^2 = 0$

$(c^2 - a^2)x^2 - a^2y^2 = a^2c^2 - a^4$

$(c^2 - a^2)x^2 - a^2y^2 = a^2(c^2 - a^2)$

$c^2 - a^2 = b^2$

$x^2 - \frac{a^2 y^2}{c^2 - a^2} = a^2$

$\frac{x^2}{a^2} - \frac{y^2}{c^2 - a^2} = 1$

$c^2 - a^2 = b^2$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

EQ IPERBOLE CON FUOCHI ASSE X

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$$

EQ IPERBOLE CON FUOCHI ASSE Y

$c^2 - b^2 = a^2$

$c^2 - a^2 = b^2$