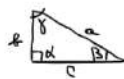


Lezione 26



$$c = 7$$

$$\gamma = 15^\circ$$

$$\alpha = 90^\circ$$

$$c = a \cdot \sin \gamma \Rightarrow a = \frac{c}{\sin \gamma}$$

$$a = \frac{7}{\sin 15^\circ} = \frac{7}{\frac{\sqrt{6}-\sqrt{2}}{4}} = \frac{28(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})(\sqrt{6}+\sqrt{2})}$$

$$= \frac{28\sqrt{6} + 28\sqrt{2}}{6-2} = 7 \cdot 7(\sqrt{6}+\sqrt{2})$$

$$a = 7(\sqrt{6} + \sqrt{2})$$

$$b = a \cos \gamma = 7(\sqrt{6} + \sqrt{2}) \cos 15^\circ$$

$$\sin(15^\circ) = \sin(60^\circ - 45^\circ) =$$

$$= \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ =$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos 15^\circ = \cos(60^\circ - 45^\circ) =$$

$$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ =$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

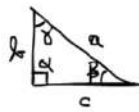
$$b = 7(\sqrt{6} + \sqrt{2}) \cdot \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$b = \frac{7}{4}(6 + 2 + 2\sqrt{12}) = \frac{7}{4}(8 + 4\sqrt{3}) =$$

$$= 7 \cdot 2 + 7\sqrt{3} = 7(2 + \sqrt{3})$$

$$\beta = 180^\circ - (\alpha + \gamma) = 180^\circ - (90^\circ + 15^\circ) =$$

$$= 180^\circ - 105^\circ = 75^\circ$$



$$b = \sqrt{6} + 2\sqrt{2}$$

$$c = \sqrt{2}$$

$$\alpha = 90^\circ$$

$$b = a \sin \beta \Rightarrow a = \frac{b}{\sin \beta}$$

$$c = a \cos \beta \Rightarrow a = \frac{c}{\cos \beta}$$

$$\frac{b}{\sin \beta} = \frac{c}{\cos \beta} \Rightarrow \frac{b}{c} = \frac{\sin \beta}{\cos \beta} = \tan \beta$$

$$\tan \beta = \frac{b}{c} = \frac{\sqrt{6} + 2\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}(\sqrt{3} + 2)}{\sqrt{2}}$$

$$\tan \beta = 2 + \sqrt{3} \Rightarrow \beta = \arctan(2 + \sqrt{3}) = 75^\circ$$

$$\gamma = 180^\circ - (\alpha + \beta) = 180^\circ - (90^\circ + 75^\circ) = 180^\circ - 165^\circ = 15^\circ$$

$$b = a \sin \beta \Rightarrow a = \frac{b}{\sin \beta} = \frac{\sqrt{6} + 2\sqrt{2}}{\sin 75^\circ}$$

$$= \frac{\sqrt{6} + 2\sqrt{2}}{\frac{\sqrt{6} + \sqrt{2}}{4}} = \frac{4(\sqrt{6} + 2\sqrt{2})}{\sqrt{6} + \sqrt{2}} \cdot \frac{(\sqrt{6} - \sqrt{2})}{\sqrt{6} - \sqrt{2}}$$

$$= \frac{4(6 - \sqrt{12} + 2\sqrt{12} - 4)}{4} = 2 + \sqrt{12} = 2 + 2\sqrt{3} = 2(1 + \sqrt{3})$$

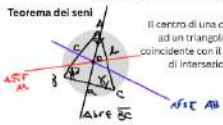
$$b = 2 + \sqrt{12} = 2 + 2\sqrt{3} = 2(1 + \sqrt{3})$$

$$\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$$

Teoremi sui triangoli qualunque

Teorema dei seni

Il centro di una circonferenza circoscritta ad un triangolo qualunque è sempre coincidente con il circocentro, cioè il punto di intersezione degli assi dei lati.



L'asse di un lato è quella retta perpendicolare al lato passante per il suo punto medio.

O CIRCOCENTRO

$$\begin{matrix} \overline{AB} = c \\ \overline{BC} = a \\ \overline{AC} = b \end{matrix}$$

CORDE

Teorema della corda (\*\*\*)

Dimm



$$\frac{\overline{AB}}{\sin \widehat{ACB}} = 2R$$

BC, DIAMETRO

$$\widehat{ACB} = \frac{1}{2} \widehat{AOB}$$

$$\widehat{AOB} = \frac{1}{2} \widehat{AOB}$$

$$\widehat{ACB} = \widehat{AOB}$$

BAC



Questo triangolo è inscritto in una semicirconferenza e quindi è rettangolo

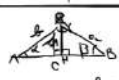
$$\overline{AB} = \overline{BC} \cdot \sin \widehat{ACB}$$

$$\frac{\overline{AB}}{\sin \widehat{ACB}} = \overline{BC} = 2R$$

C. V. d

$$\frac{a}{\sin \alpha} = 2R \quad \frac{b}{\sin \beta} = 2R \quad \frac{c}{\sin \gamma} = 2R$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$



$$h = a \sin \beta$$

$$h = b \sin \alpha$$

$$a \sin \beta = b \sin \alpha$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$



$$\begin{aligned} a &= 4 \\ b &= 4\sqrt{2} \\ \beta &= 45^\circ \end{aligned}$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$\frac{4}{\sin \alpha} = \frac{4\sqrt{2}}{\sin 45^\circ}$$

$$\frac{4}{\sin \alpha} = \frac{4\sqrt{2}}{\frac{\sqrt{2}}{2}} \quad \frac{1}{\sin \alpha} = 2$$

$$\sin \alpha = \frac{1}{2}$$

$$\alpha = \arcsin \frac{1}{2} = 30^\circ$$

$$\begin{aligned} \gamma &= 180^\circ - (\alpha + \beta) = 180^\circ - (30^\circ + 45^\circ) \\ &= 180^\circ - 75^\circ = \boxed{105^\circ} \end{aligned}$$

TEOR DE SEN

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma}$$

$$\frac{4}{\sin 30^\circ} = \frac{c}{\sin 105^\circ}$$

$$\frac{4}{\frac{1}{2}} = \frac{c}{\frac{\sqrt{6} + \sqrt{2}}{4}} \Rightarrow c = \frac{4c}{\sqrt{6} + \sqrt{2}}$$

$$c = \frac{2(\sqrt{6} + \sqrt{2})}{1}$$

$$\boxed{c = 2(\sqrt{6} + \sqrt{2})}$$