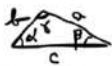


Lezione 30

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$



Teorema dei seni



$$\begin{cases} a = 4\sqrt{3} \\ b = 4 \\ c = 4 \end{cases}$$



150°

$$\gamma = \beta$$

$\sin \gamma = \sin \beta$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

$$\begin{cases} \frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \\ \frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \\ \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \end{cases}$$

$$\begin{cases} \frac{4\sqrt{3}}{\sin \alpha} = \frac{4}{\sin \beta} \\ \frac{4\sqrt{3}}{\sin \alpha} = \frac{4}{\sin \gamma} \\ \frac{4}{\sin \beta} = \frac{4}{\sin \gamma} \end{cases}$$

$$\frac{4\sqrt{3}}{\sin \alpha} = \frac{4}{\sin \beta}$$

$$\frac{4\sqrt{3}}{\sin \alpha} = \frac{4}{\sin \beta}$$

$$\frac{4}{\sin \beta} = \frac{4}{\sin \beta} \Rightarrow 1 = 1$$

$$\frac{4\sqrt{3}}{\sin \alpha} = \frac{4}{\sin \beta}$$

$$\sin \beta = \frac{1}{\sqrt{3}} \sin \alpha$$

???

CON CANNOT !!!

$$a = 2\sqrt{3} + 3$$

$$\begin{aligned} \beta &= 15^\circ \\ \gamma &= 90^\circ \\ \alpha &= 75^\circ \end{aligned}$$



$$\alpha = 180^\circ - (\gamma + \beta) = 180^\circ - 105^\circ = 75^\circ$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \Rightarrow b = a \cdot \frac{\sin \beta}{\sin \alpha}$$

$$b = (2\sqrt{3} + 3) \cdot \frac{\sin 15^\circ}{\sin 75^\circ}$$

$$b = (2\sqrt{3} + 3) \cdot \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} = (2\sqrt{3} + 3) \cdot \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}}$$

$$\frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} + \sqrt{2}} \cdot \frac{(\sqrt{6} - \sqrt{2})}{(\sqrt{6} - \sqrt{2})} = \frac{6 + 2 - 2\sqrt{12}}{6 - 2} = \frac{8 - 2\sqrt{12}}{4}$$

$$= \frac{8 - 2 \cdot 2\sqrt{3}}{4} = \frac{8 - 4\sqrt{3}}{4} = \frac{2 - \sqrt{3}}{1}$$

$$b = (2\sqrt{3} + 3)(2 - \sqrt{3}) = 4\sqrt{3} - 6 + 6 - 3\sqrt{3} = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$$

$$\frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \Rightarrow c = b \cdot \frac{\sin \gamma}{\sin \beta} \quad \boxed{b = \sqrt{3}}$$

$$c = \sqrt{3} \cdot \frac{\sin 90^\circ}{\sin 15^\circ} = \sqrt{3} \cdot \frac{1}{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

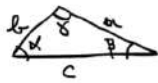
$$c = 4\sqrt{3} \cdot \frac{1}{\sqrt{6} - \sqrt{2}} = \frac{4\sqrt{3}}{\sqrt{6} - \sqrt{2}} = \frac{4\sqrt{3}}{6 - 2} = \frac{4\sqrt{3}}{4}$$

$$c = \sqrt{3} \cdot \frac{(\sqrt{2} + \sqrt{2})}{\sqrt{2} + \sqrt{2}} = \sqrt{3}(\sqrt{2} + \sqrt{2})$$

$$= \sqrt{18} + \sqrt{6} = 3\sqrt{2} + \sqrt{6}$$

$$\boxed{c = 3\sqrt{2} + \sqrt{6}}$$

$$\begin{array}{r} 18 \\ 6 \\ 3 \\ 3 \\ 1 \end{array}$$



$$\begin{aligned} a &= 2 \\ \alpha &= 45^\circ \\ \beta &= 75^\circ \\ \gamma &= 60^\circ \end{aligned}$$

$$\gamma = 180^\circ - (\alpha + \beta) = 180^\circ - 120^\circ = 60^\circ$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} \Rightarrow b = a \cdot \frac{\sin \beta}{\sin \alpha}$$

$$b = 2 \cdot \frac{\sin 75^\circ}{\sin 45^\circ} = 2 \cdot \frac{\frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{\sqrt{2}}{2}}$$

$$b = \cancel{2} \cdot \frac{\sqrt{6} + \sqrt{2}}{\cancel{4}} \cdot \frac{\cancel{2}}{\sqrt{2}} = \frac{\sqrt{6} + \sqrt{2}}{\sqrt{2}}$$

$$b = \frac{\sqrt{2}(\sqrt{3} + 1)}{\sqrt{2}} = \frac{1 + \sqrt{3}}{\cancel{2}} \quad \boxed{b = 1 + \sqrt{3}}$$

$$\frac{a}{\sin \alpha} = \frac{c}{\sin \gamma} \Rightarrow c = a \cdot \frac{\sin \gamma}{\sin \alpha}$$

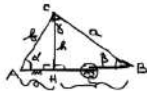
$$c = 2 \cdot \frac{\sin 60^\circ}{\sin 45^\circ} = 2 \cdot \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{2}}{2}} = 2 \cdot \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{2}} \quad |$$

$$c = 2 \cdot \frac{\sqrt{3}}{\sqrt{2}} = \cancel{2} \cdot \frac{\sqrt{6}}{\cancel{2}}$$

$$\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

$$\boxed{c = \sqrt{6}}$$

Teorema dei coseni e di Carnot



$$\overline{AH} = m$$

$$\overline{BH} = n$$

AHC e HCB TRIANGOLI RETTANGOLI

$$a^2 = h^2 + m^2$$

NEL TRIANGOLO HCB

$$h = b \sin \alpha$$

$$m = b \cos \alpha$$

TRIANGOLO ACH

TRIANGOLO ACH

$$m = c - n$$

$$\overline{BH} = \overline{AB} - \overline{AH}$$

$$m = c - b \cos \alpha$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$a^2 = h^2 + m^2$$

$$a^2 = (b \sin \alpha)^2 + (c - b \cos \alpha)^2$$

$$a^2 = b^2 \sin^2 \alpha + c^2 + b^2 \cos^2 \alpha - 2bc \cos \alpha$$

$$a^2 = b^2 (\sin^2 \alpha + \cos^2 \alpha) + c^2 - 2bc \cos \alpha$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

QUADRO DI C  
 DOTTI PROPRIETÀ NEGATIVE DI b e c  
 Moltiplicato il coseno dell'angolo compreso fra b e c  
 $\cos 90^\circ = 0$   
 $a^2 = b^2 + c^2$  PITAGORAT



$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$



$$b = c = 4$$

$$a = 4\sqrt{3}$$

$$\boxed{a^2 = b^2 + c^2 - 2bc \cos \alpha}$$

$$(4\sqrt{3})^2 = 16 + 16 - 2 \cdot 16 \cos \alpha$$

$$48 = 32 - 32 \cos \alpha$$

$$16 = -32 \cos \alpha \implies \cos \alpha = -\frac{16}{32} = -\frac{1}{2}$$



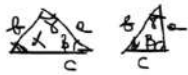
$$\frac{180^\circ}{2} = 90^\circ$$

$$\cos \alpha = -\frac{1}{2} \implies \alpha = \arccos\left(-\frac{1}{2}\right)$$

$$\boxed{\alpha = 120^\circ}$$

$$\gamma = \beta = \frac{180^\circ - \alpha}{2} = \frac{180^\circ - 120^\circ}{2} = \frac{60^\circ}{2} = 30^\circ$$

$$\boxed{\gamma = 30^\circ}$$



$$a = \sqrt{3}$$

$$b = 2$$

$$\gamma = 30^\circ$$

$$\begin{cases} a^2 = b^2 + c^2 - 2bc \cos \alpha \\ b^2 = a^2 + c^2 - 2ac \cos \beta \\ c^2 = a^2 + b^2 - 2ab \cos \gamma \end{cases}$$

$c, \cos \alpha, \cos \beta$   
 $\cos 30^\circ = \frac{\sqrt{3}}{2}$

$$c^2 = (\sqrt{3})^2 + (2)^2 - 2 \cdot \sqrt{3} \cdot 2 \cos 30^\circ$$

$$c^2 = 3 + 4 - \frac{2 \cdot \sqrt{3} \cdot \sqrt{3}}{2} = 7 - 6 = 1$$

$$c^2 = 1 \Rightarrow \boxed{c = 1}$$

$$\begin{cases} a^2 = b^2 + c^2 - 2bc \cos \alpha \\ b^2 = a^2 + c^2 - 2ac \cos \beta \end{cases}$$

$$\begin{cases} 3 = 4 + 1 - 2 \cdot 2 \cdot \cos \alpha \\ 4 = 3 + 1 - 2 \cdot \sqrt{3} \cos \beta \end{cases}$$

$$\begin{cases} 3 = 5 - 4 \cos \alpha \Rightarrow \cos \alpha = \frac{1}{2} \\ 4 = 4 - 2\sqrt{3} \cos \beta \Rightarrow 2\sqrt{3} \cos \beta = 0 \end{cases}$$

$\cos \alpha = \frac{1}{2}$   
 $\alpha = 60^\circ$   
 $\cos \beta = 0$   
 $\beta = 90^\circ$

$$\beta = 180^\circ - (\alpha + \gamma) = 180^\circ - (60^\circ + 30^\circ) = 90^\circ$$

$a = \sqrt{3}$	$\alpha = 60^\circ$
$b = 2$	$\beta = 90^\circ$
$c = 1$	$\gamma = 30^\circ$