Lezione 28

Integrali delle funzioni goniometriche

(1) $\boxed{\int \cos x \, dx = \operatorname{sen} x + C}$

(2) $\boxed{\int \operatorname{sen} x \, dx = -\cos x + C}$

(3) $\boxed{\int \frac{1}{\cos^2 x} \, dx = \operatorname{tg} x + C}$

(4) $\boxed{\int \frac{1}{\operatorname{sen}^2 x} \, dx = -\operatorname{cotg} x + C}$

$$\operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x}$$

$$\frac{d \operatorname{tg} x}{dx} = \frac{\cos x \cos x - \operatorname{sen} x (-\operatorname{sen} x)}{\cos^2 x} = \frac{\cos^2 x + \operatorname{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\frac{d \operatorname{cotg} x}{dx} = \frac{-\operatorname{sen} x \cdot \operatorname{sen} x - \cos x \cdot \cos x}{\operatorname{sen}^2 x} = -\frac{\operatorname{sen}^2 x + \cos^2 x}{\operatorname{sen}^2 x} = -\frac{1}{\operatorname{sen}^2 x}$$

$\Downarrow$

1' $\boxed{\int \cos f(x) \cdot f'(x) = \operatorname{sen} f(x) + C}$

2' $\boxed{\int \operatorname{sen} f(x) \cdot f'(x) = -\cos f(x) + C}$

3' $\boxed{\int \frac{f'(x)}{\cos^2 f(x)} \, dx = \operatorname{tg} f(x) + C}$

4' $\boxed{\int \frac{f'(x)}{\operatorname{sen}^2 f(x)} \, dx = -\operatorname{cotg} f(x) + C}$

$$D_x(\operatorname{arcsen} x) = \frac{1}{\sqrt{1-x^2}} \Longrightarrow \int \frac{1}{\sqrt{1-x^2}}\,dx = \operatorname{arcsen} x + c$$

$$D_x(\operatorname{arcsen} f(x)) = \frac{f'(x)}{\sqrt{1-[f(x)]^2}} \Longrightarrow \int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}\,dx = \operatorname{arcsen} f(x) + C$$

$$D_x(\arccos x) = -\frac{1}{\sqrt{1-x^2}} \Longrightarrow \int \frac{1}{\sqrt{1-x^2}}\,dx = -\arccos x$$

$$D_x(\arccos f(x)) = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}} \Longrightarrow \int \frac{f'(x)}{\sqrt{1-[f(x)]^2}}\,dx = -\arccos f(x) + C$$

$$D_x(\operatorname{arctg} x) = \frac{1}{1+x^2} \Longrightarrow \int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x + c$$

$$D_x(\operatorname{arctg} f(x)) = \frac{f'(x)}{1+[f(x)]^2} \Longrightarrow \int \frac{f'(x)}{1+[f(x)]^2}\,dx = \operatorname{arctg} f(x) + C$$

$m \in \mathbb{R}$

$$\int \frac{1}{x^2+m^2}\,dx = \int \frac{1}{m^2\left(\frac{x^2}{m^2}+1\right)}\,dx =$$

$f(x) = \frac{x}{m}$

$$\frac{1}{m^2}\int \frac{1}{1+\left(\frac{x}{m}\right)^2}\,dx =$$

$f'(x) = \frac{1}{m}$

$$= \frac{1}{m^2}\, m \int \frac{\frac{1}{m}}{1+\left(\frac{x}{m}\right)^2}\,dx = \boxed{\frac{1}{m}\operatorname{arctg}\frac{x}{m} + c}$$

$$\boxed{\int \frac{1}{x^2+m^2}\,dx = \frac{1}{m}\operatorname{arctg}\frac{x}{m} + c}$$

$$\boxed{\int \frac{f'(x)}{[f(x)]^2+m^2}\,dx = \frac{1}{m}\operatorname{arctg}\frac{f(x)}{m} + c}$$

Integrali delle funzioni esponenziali

$$\boxed{\int e^x\,dx = e^x + c}$$

$$\boxed{\int e^{f(x)} f'(x)\,dx = e^{f(x)} + c}$$

$$\boxed{\int a^x\,dx = \frac{a^x}{\log a} + c}$$

$0<a<1$
$a>1$

$$\boxed{\int a^{f(x)} \cdot f'(x)\,dx = \frac{a^{f(x)}}{\log a} + c}$$

$D_x(e^x) = e^x$

$D_x(e^{f(x)}) =$
$e^{f(x)} \cdot f'(x)$

$D_x\left(\frac{a^x}{\log a}\right) = a^x$

$\boxed{D_x a^x = a^x \cdot \log a}$

$D_x(a^{f(x)}) =$
$= a^{f(x)} \cdot \log a \cdot f'(x)$