Lezione 30

$$\int \frac{1}{x^3-6x^2+11x-6}\,dx =$$

$P(x): x^3-6x^2+11x-6$

$P(1) = 1-6+11-6 = -5+5=0$ $\quad D_6 = \{\pm1, \pm2, \pm3, \pm6\}$

$$\begin{array}{c|ccc|c} & 1 & -6 & 11 & -6 \\ 1 & & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

$(x-1)$

$x^3-6x^2+11x-6 = (x-1)(x^2-5x+6)$

$x^2-5x+6 =$
$= (x-2)(x-3)$

$P=6$
$S=-5$

$1 \cdot 6$
$2 \cdot 3$ $\quad -2 \quad -3$

$x^3-6x^2+11x-6 = (x-1)(x-2)(x-3)$

$$\int \frac{1}{x^3-6x^2+11x-6}\,dx = \int \frac{1}{(x-1)(x-2)(x-3)}\,dx$$

$$\frac{1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$\frac{1}{(x-1)(x-2)(x-3)} = \frac{A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)}{(x-1)(x-2)(x-3)}$$

$1 = A(x^2-5x+6)+B(x^2-4x+3)+C(x^2-3x+2)$

$1 = (A+B+C)x^2 + (-5A-4B-3C)x + 6A+3B+2C$

$$\begin{cases} A+B+C=0 \\ -5A-4B-3C=0 \Rightarrow 5A+4B+3C=0 \\ 6A+3B+2C=1 \end{cases}$$

$$\begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}\begin{cases} A+B+C=0 \\ 5A+4B+3C=0 \\ 6A+3B+2C=1 \end{cases} \Rightarrow \begin{array}{l} 3R_1 \\ R_2 \\ R_3 \end{array}\begin{cases} 3A+3B+3C=0 \\ 5A+4B+3C=0 \\ 6A+3B+2C=1 \end{cases}$$

$$\begin{array}{l} R_1 \\ 3R_1-R_2 \\ R_3 \end{array}\begin{cases} A+B+C=0 \\ -2A-B=0 \\ 6A+3B+2C=1 \end{cases} \Rightarrow \begin{array}{l} 2R_1 \\ 3R_1-R_2 \\ R_3 \end{array}\begin{cases} 2A+2B+2C=0 \\ -2A-B=0 \\ 6A+3B+2C=1 \end{cases}$$

$$\begin{array}{l} R_1 \\ 3R_1-R_2 \\ 2R_1-R_3 \end{array}\begin{cases} A+B+C=0 \\ -2A-B=0 \\ -4A-B=-1 \end{cases} \qquad \begin{array}{l} R_1' \\ R_2' \end{array}\begin{cases} -2A-B=0 \\ -4A-B=-1 \end{cases}$$

$R_1'-R_2' \quad 2A=1$

$$\boxed{A=\frac{1}{2}}$$

$2A+B=0$

$\cancel{2}\frac{1}{\cancel{2}}+B=0 \Rightarrow \boxed{B=-1}$

$A+B+C=0$

$C=1-\frac{1}{2} \Rightarrow \boxed{C=\frac{1}{2}}$ $\quad \begin{cases} A=\frac{1}{2} \\ B=-1 \\ C=\frac{1}{2} \end{cases}$

$$\int \frac{\frac{1}{2}}{x-1}\,dx + \int \frac{-1}{x-2}\,dx + \int \frac{\frac{1}{2}}{x-3}\,dx$$

$= \frac{1}{2}\log|x-1| - 1\log|x-2| + \frac{1}{2}\log|x-3| + c$

$$= \boxed{\frac{1}{2}\left(\log|x-1| + \log|x-3|\right) - \log|x-2| + c}$$

$= \log\sqrt{|x-1||x-3|} - \log|x-2| + c =$

$$= \boxed{\log\frac{\sqrt{|x-1||x-3|}}{|x-2|} + c}$$

$$\int \frac{1}{x^3+2x^2+x+2}\,dx$$

$$x^3+2x^2+x+2 = x^2(x+2)+1(x+2) =$$

$$= (x+2)(x^2+1)$$

$$\frac{1}{x^3+2x^2+x+2} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$\frac{1}{\cancel{x^3+2x^2+x+2}} = \frac{A(x^2+1)+(Bx+C)(x+2)}{\cancel{(x+2)(x^2+1)}}$$

$$1 = Ax^2 + A + Bx^2 + 2Bx + Cx + 2C$$

$$1 = (A+B)x^2 + (2B+C)x + A+2C$$

$$\begin{cases} A+B=0 \\ 2B+C=0 \\ A+2C=1 \end{cases}$$

$$\begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}\begin{cases} A+B \quad = 0 \\ \quad 2B+C = 0 \\ A \quad +2C = 1 \end{cases} \Longrightarrow \begin{matrix} R_1 \\ R_2 \\ R_1-R_3 \end{matrix}\begin{cases} A+B \quad = 0 \\ 2B+C = 0 \\ B-2C=-1 \end{cases}$$

$$\begin{matrix} R_1' \\ R_2' \end{matrix}\begin{cases} 2B+C=0 \\ B-2C=-1 \end{cases} \quad \begin{matrix} R_1' \\ 2R_2' \end{matrix}\begin{cases} 2B+C=0 \\ 2B-4C=-2 \end{cases}$$

$$R_1'-2R_2' \quad 5C=2 \quad \boxed{C=\frac{2}{5}}$$

$$B-2C=-1$$

$$B-2\cdot\frac{2}{5}=-1 \quad B-\frac{4}{5}=-1$$

$$B=\frac{4}{5}-1=\frac{4-5}{5}=-\frac{1}{5} \quad \boxed{B=-\frac{1}{5}}$$

$$A+B=0 \Longrightarrow \boxed{A=\frac{1}{5}}$$

$$\int \frac{\frac{1}{5}}{x+2}\,dx + \int \frac{-\frac{1}{5}x+\frac{2}{5}}{x^2+1}\,dx$$

$$=\frac{1}{5}\log|x+2| + \int \frac{-\frac{1}{5}(x-2)}{x^2+1}\,dx$$

$$=\frac{1}{5}\log|x+2| - \frac{1}{5}\int \frac{x-2}{x^2+1}\,dx = \qquad \int \frac{f'(x)}{f(x)}\,dx$$

$$=\frac{1}{5}\log|x+2| - \frac{1}{5}\frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \frac{2}{5}\int \frac{1}{x^2+1}\,dx$$

$$=\frac{1}{5}\log|x+2| - \frac{1}{10}\log|x^2+1| + \frac{2}{5}\operatorname{arctg} x + c =$$

$$=\frac{1}{5}\log|x+2| - \frac{1}{5}\cdot\frac{1}{2}\log|x^2+1| + \frac{2}{5}\operatorname{arctg} x + c =$$

$$=\frac{1}{5}\log\frac{|x+2|}{\sqrt{|x^2+1|}} + \frac{2}{5}\operatorname{arctg} x + C \qquad \boxed{\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x}$$

Integrazione per sostituzione

Sostituiamo una variabile più "scomoda" con una più comoda da integrare!!!

$$\boxed{\int f(x)\,dx = \int f(g(t))\,g'(t)\,dt}$$

$$\boxed{\begin{aligned} x &= g(t) \\ dx &= \frac{dg(t)}{dt}\,dt \end{aligned}} \quad \to g'(t)$$

<u>ES</u> $\int \sqrt{3x+2}\,dx = \int (3x+2)^{\frac{1}{2}}\,dx =$

$$= \frac{1}{3}\int 3\cdot(3x+2)^{\frac{1}{2}}\,dx =$$

$$= \frac{1}{3}\cdot\frac{(3x+2)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{1}{3}\cdot\frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \boxed{\frac{2}{9}\sqrt[3]{(3x+2)^2} + C}$$

$$\int f(x)^{\alpha}\cdot f'(x)\,dx = \frac{f(x)^{\alpha+1}}{\alpha+1}$$

$\boxed{\text{INTEGR. IMMEDIATA}}$

$$\int \sqrt{3x+2}\,dx$$

$$\boxed{3x+2=t}$$

$$3x = t-2$$

$$x = \frac{t-2}{3} \to g(t)$$

$\boxed{\text{SOSTITUZ.}}$

$$\boxed{dx = g'(t)\,dt = \frac{1}{3}\,dt}$$

$$= \int \sqrt{t}\cdot\frac{1}{3}\,dt =$$

$$= \frac{1}{3}\int\sqrt{t}\,dt = \frac{1}{3}\int t^{\frac{1}{2}}\,dt = \frac{1}{3}\,\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= \frac{2}{9}\,(2x+3)^{\frac{3}{2}} + C = \boxed{\frac{2}{9}\sqrt[3]{(2x+3)^2} + C}$$

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx =$$

$$= \int \frac{\cos t}{\cancel{t}} \cdot \cancel{2}t\,dt =$$

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$$\boxed{\begin{array}{l} \sqrt{x} = t \qquad g(t)=t^2 \\ x = t^2 \\ dx = g'(t)\,dt = 2t\,dt \end{array}}$$

$$= 2\int \cos t\,dt = 2\,\mathrm{sen}\,t + c$$

$$\boxed{\text{SOSTITUZ}}$$

$$= 2\,\mathrm{sen}\sqrt{x} + c$$

$$\boxed{\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}}$$

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int \cos\sqrt{x} \cdot \frac{1}{2\sqrt{x}}\,dx$$

$$= 2\,\mathrm{sen}\sqrt{x} + c$$

$$\int \cos f(x)\, f'(x)\,dx$$
$$= \mathrm{sen}\,f(x) + c$$

$$\boxed{\text{IMMEDIATA}}$$

$$t = \sqrt{e^x - 4}$$

$$e^x - 4 = t^2$$

$$e^x = t^2 + 4$$

$$x = \log_e(t^2+4) \to g(t)$$

$$dx = g'(t)\,dt = \frac{1}{t^2+4}\cdot 2t\,dt$$

$$\int \sqrt{e^x - 4}\,dx =$$

$$= \int \sqrt{t^2}\,\frac{2t}{t^2+4}\,dt =$$

$$= \int \frac{t \cdot 2t}{t^2+4}\,dt =$$

$$= \int \frac{2t^2}{t^2+4}\,dt = (2)\int \frac{t^2}{t^2+4}\,dt = \int \frac{t^2+4-4}{t^2+4}\,dt =$$

$$= \int \cancel{\frac{t^2+4}{t^2+4}}\,dt - \int \frac{4}{t^2+4}\,dt =$$

$$= \int dt - 4\int \frac{1}{t^2+4}\,dt =$$

$$= t - \cancel{4}\int \frac{1}{\cancel{4}\left(1+\frac{t^2}{4}\right)}\,dt$$

$$\int \frac{f'(x)}{1+[f(x)]^2}\,dx = \operatorname{arctg} f(x) + c$$

$$= t - \int \frac{1}{1+\left[\frac{t^2}{4}\right]}\,dt =$$

$$= t - 2\int \frac{1\cdot\frac{1}{2}}{1+\left[\frac{t}{2}\right]^2}\,dt = t - 2\operatorname{arctg}\left(\frac{t}{2}\right) + c$$

$$\int \log x\,dx$$

$$* = 2t - 4\operatorname{arctg}\left(\frac{t}{2}\right) + c =$$

$$= 2\sqrt{e^x-4} - 4\operatorname{arctg}\left(\frac{\sqrt{e^x-4}}{2}\right) + c$$